

THE LOGIC OF DECISION*

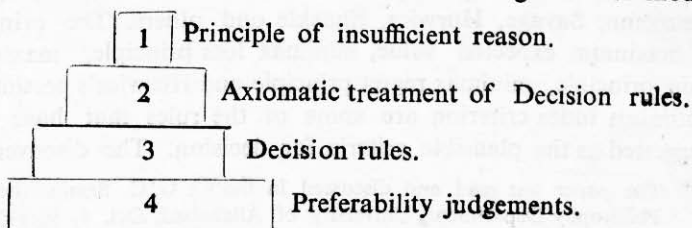
I

The term 'logic of decision' may be used in several senses. At first, it may mean a single set of reasons for a particular decision. For example, suppose a man, M, decides to vote for A rather than for B and gives a reason for this decision that A is more liberal than B or A will offer him more services than B, then we may say that M has a logic of decision. But suppose, again, that M is ultimately persuaded by C to vote for B rather than for A on the ground that B belongs to M's close relations, or caste or community although he is neither liberal nor is he to offer any services to M. Then also it will be said that M has a logic of decision. In these two contexts the term 'logic' means a set of reasons which may be valid or invalid but which must be convincing to the decision-maker. He may have any reasons for his decision or he may decide irrationally and later on give any reasons for his decision. So the question arises, should logic be used in this sense of a set of reasons? Obviously logic is more than a set of reasons and to use it in this sense is a very narrow use of the term 'logic'. Moreover, this use is also fallacious for it may raise a set of fallacious reasons to the status of logically valid reasons. All reasons are not logical deductions although all logical deductions are reasons. Moreover, if this use is permitted, M can have more than one logic of decision and these logics may be mutually contrary if not contradictory. So the use of the term 'logic of decision' in the sense of a set of reasons, cooked or conceived *ad libitum* is an abuse of language.

Secondly, the logic of decision may mean a principle on the basis of which decision-makers, more often than not, exercise their decision. Such principles have been advanced by Bayes, von Neumann, Savage, Hurwicz, Shackle and others. The principle of maximum expected value, minimax loss principle, maximum gain principle, minimax regret principle and Hurwicz's pessimism-optimism index criterion are some of the rules that have been suggested as the plausible criteria for decision. The discovery of

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these decision criteria has doubtless exhibited the fact that there are rules of decision which are followed knowingly or unknowingly and which make decision making a rational state of affairs. But none of these decision criteria is universally applicable although all of them are involved in local decision-making. So attempts have been made, particularly by Chernoff and Milnor, to arrive at certain desiderata which fulfil decision criteria. Chernoff's 11 axioms and Milnor's 10 axioms¹ are the two important attempts for stating a set of requirements for reasonable criteria. On the basis of these requirements it has been found that none of the above mentioned decision criteria is satisfactory. So if the term 'logic of decision' is used in the sense of decision criteria, it will not include the axiomatic treatment of these criteria, which is obviously more logical than the framing of a decision criterion that is derived more or less from intuitions, conjectures or trial and error methods. Logic does not deal with how to reason or decide: it deals with the correct reasoning or deciding. Its function is the verification or falsification of reasons that are given. At any rate the logic of decision is not an application of the logic of scientific discovery in the field of social sciences or decision-making. Furthermore, even the axiomatic treatment of Chernoff and Milnor requires further abstraction and generalization inasmuch as it has a logical bearing on the principle of insufficient reason which has been variously interpreted in the literature on decision theory. It is not impossible to show that the axioms testing decision criteria are deducible from the principle of insufficient reason which appears to have the same status in the logic of decision as the Laws of Thought have in the general field of logic. So even the axiomatic treatment of decision criteria cannot be identified with the main domain of the logic of decision, although it is doubtless a part of this logic. The full scope of the logic of decision can now be represented in the following four-tier model.



Now this four-tier model includes all decision judgements that are made under uncertainty or risk. Furthermore, if we use a disjunction at its apex, i.e., we add some more principle or principles along with the principle of insufficient reason, it can include even those decision judgements which are made under certainty. Savage's doctrine of sure things principle or strong Independence Assumption is one such principle.

Thirdly, R. C. Jefferey who has written a book entitled *The Logic of Decision* has used the term in the sense of "a framework within which one can study the relations among various possible belief, value and desirability functions and between these and policies for decision-making"². For him this framework is "normative in much the same way as deductive logic" and is put forth "as a useful representation of some very general norms for the formulation and critique of belief and decision."³ He has, by and large, developed his concept of the logic of decision from the Bayesian account of deliberation and excluded from it such disciplines as statistics, inductive logic and ethics. What makes his concept too narrow is the exclusion of inductive logic from it. He has made it an application of symbolic logic and forgotten that the logic of decision is more linked with inductive logic than with deductive logic. A number of logicians especially I. Levi, J. Hintikka and H. E. Kyburg have come forward who have studied the problem of decision-making from the stand-point of inductive logic. They have discovered a new domain of inductive logic which is called local induction, and applied decision theory to the problems of inductive logic. Three major accomplishments in this field are listed by Willaim K. Goosens. They are as follows:⁴

1. Decision theory can be applied to the problem of evaluating hypotheses in a manner which is compelling.
2. Hypotheses can be accepted in such a way that closure conditions are satisfied.
3. The concept of an epistemic interest can be clarified and defended.

These researches are logically more significant than Jefferey's reduction of the logic of decision to a form of deductive logic. Moreover, Jefferey has elucidated the notions of subjective probability and subjective desirability or utility by means of elementary

logical operations such as denial of conjunction and disjunction. His elucidation is significant in the sense that it has represented the Bayesian theory in a logical form that can be understood by the students of logic. But he has used the term 'the logic of decision' in a limited sense and for a limited purpose. His elucidation has not made any significant development of the logic of decision itself. It seems that for him the logic of decision is an extension or application of deductive logic. But decision is more than deductive reasoning and the logic of decision is not an extension of deductive logic. Nicholas Rescher rightly points out that "theoretical logic alone is not in a position to tell us more than that one must make choices : it will not—indeed cannot—tell us what particular choices are to be made."⁵

Fourthly, Nicholas Rescher himself thinks that the logic of decision is an applied logic. He calls it "epistemic methodology" which uses the tools of theoretical logic with due extra-logical supplementation to render them capable of resolving factual issues and sketches at a high level of abstracts and generality of the epistemic considerations that bear upon the rationally warranted determination of the truth-status of propositions concerning decision making. In short it deals with preferability judgements and the rules that govern them. Now it will be found that Nicholas Rescher has used the logic of decision in an ambiguous sense. His concept when analysed will yield the following elements.

(i) an epistemic methodology of high level of abstract generalization, (ii) use of the rules of deductive logic along with extra-logical considerations to resolve the factual issues and (iii) a rational or logical treatment of preferability judgements. As the first of these elements suggests, the term 'logic' has been used in a very general sense of concept formation. The second element makes the logic of decision an applied logic and the third element gives it a new domain of preferability judgements. In fact Nicholas Rescher has tried to survey the whole field of the logic of decision. But his survey is only a description. It is not definition in as-much as it does not indicate the essential nature of the logic of decision and mixes its essential nature with the non-essential ones. So there is still a scope for determining the nature of the logic of decision. Thus we come to the fifth view : *the logic of decision is the logic of preferability judgements which are made under*

the conditions of certainty, uncertainty or risk. In order to elaborate this view we have to classify some concepts that have gathered around it.

II

A question is raised whether the logic of decision is deductive logic or inductive logic. The logic of decision is the logic of uncertainty in as much it deals with decisions that are made, more often than not, under uncertainty. So it resembles the logic of chance and/or the logic of probability. To classify it as deductive or inductive is, therefore, wrong. But if one were forced to do so it would be true to identify it with inductive logic. But it should not be then supposed that no rule of deductive logic is applicable to it. In fact the logic of decision goes beyond the dichotomy of deductive and inductive logic inasmuch as it uses the techniques of both of them. Jeffrey has shown how the rules of deductive logic can be applied to it. Further when Nicholas Rescher says that "preferability judgments are not somehow extracted *ex nihilo* by wishful thinking, but are themselves conditioned by prior knowledge or postulation,"⁶ he means that the logic of decision involves both a deductive and an inductive process. Present decisions are determined by past decisions which are themselves determined by still more past decisions and so on and so forth. In this way decision involves a *regressus ad infinitum* which may not be invariably false. Men learn from their past experiences when they deliberate. So learning and deliberation are not incompatible. Analogical reasoning and simple induction are not ruled out from the decisionary effort.⁷ Hence it is evident that the logic of decision employs both deductive and inductive logic.

III

Another question which is significant to our discussion is whether the logic of decision is pure or applied. If by pure logic we mean the logic of Pure Reason, then the logic of decision is not pure because it is the logic of practical reason. But if we accept that preference is alethic, i. e., truth-oriented and is distinguishable from the affective sort of preferences, then the logic of decision which deals with the truth-oriented preferences is pure logic. As a matter of fact, decision-making is a complex

human act that has three elements, the formal, the psychic and the inferential. The main task of the analyst of expectation is, as Shackle says, to fuse them satisfactorily.⁸ Now who is this analyst of Expected Value? Is he an economist? Is he a logician? Perhaps he is both. The *homo economicus* is a rational or prudent man. He is a rational decision-maker and the logic of decision deals with his rationality. But as he is not a Robinson Crusoe but a social man his decision is confronted, countered, supplemented, rewarded or defeated by other *hominis aeconomici*; so his decision is not unilateral. It is either bilateral or multi-lateral. So the logic of decision has to take an objective view of decision-making in a social context and study the rationales of not only individual choices but also those of social choices, the rules that govern them and the criteria that confirm or falsify them. The main function of the logic of decision is to judge the truth-status of the reasoning that is involved in all the three elements of decision-making. It is both pure logic and applied logic rolled up into one. In so far as it deals with preferability judgments, the truth-values of preference, not-preference and indifference and the various types of order relations and decidability, it is undoubtedly a kind of pure logic. But when it applies the rules of deductive and inductive logic to the discussion of these and other related problems, it becomes, a kind of applied logic. It combines probability with desirability and herein lies its uniqueness that distinguishes it from the logic of probability. It is the concrete logic of human situations.

IV

A question, again, may be raised: What is the relation between the logic of decision and the Decision Theory? Are the two identical? Or are they different from each other?

It is well known that decision theory has two main branches, normative decision theory and descriptive decision theory. The former has been formulated in classical economics, statistics, ethics, game theory, welfare economics and political theory while the latter has been advanced in experimental studies, surveys of voting behaviour, social psychology and political science.⁹ Some philosophers have also studied decision theory from a philosophical point of view and given rise to cognitive or epistemic decision

theory. Now all these ramifications of decision theory are not the same as the logic of decision. The normative decision theory is closely related to deductive logic whereas the descriptive decision theory is related to inductive logic. But the whole decision theory is related to logic in the same way as natural sciences are related to logic. It is a theory of social sciences. Just as natural sciences have a logic, social sciences have also their own logic.

The logic of decision is a preliminary form of this logic. It is uniting the social sciences on theoretical plane and initiating a meaningful dialogue among them. The interdisciplinary character of decision theory is a mark that shows its link with logic. If this link is properly developed, it will be found that there is a logic of decision which is the *ratio cognoscendi* of the decision theory that is the *ratio essendi* of the logic of decision.

The problem of decision has two philosophical ramifications, ethical and logical. Our concern here is only with the logical one which is probably more important than the ethical one inasmuch as ethical problems have logical parameters, and the pleasure-value or *preyas* or utility which is more pursued by the people of secular interests than the *summum bonum* or *Śreyas* which is the main ethical value, can better be studied by logical tools than the ethical ones. As logic is the grammar of thought, the logic of decision is the basic tool for all decision-makers. No decision theory can ignore it inasmuch as it gives clarity, precision, consistency and completeness to it. The truth-status of preferability judgments and the mode or decidability and inferability are some of the important topics that are studied by the logic of decision. The criteria that have been evolved to test the principle of decision, as for example, Milnor's criteria or Chernoff's Axioms of optimal set are the contributions of logic to the theory of decision. Moreover, the present discussion over the principle of insufficient reason as the basic principle of all axiomatizations of decision criteria is doubtless faragut with great logical significance. In formulating the principle of insufficient reason or complete ignorance we may get hints from the vast development of the principle of *ajñāna* in Advaita vedānta. At any rate it is evident that the logic of decision is different from decision theory which is to be reconstructed, modified and evaluated in its light.

V

The logic of decision is the logic of practical reason. Decision making is a central psychic event where knowing, feeling and willing are fused into one. The decision-maker has to think about the various possible acts, the events and the consequences of the act. He has to prepare a consequence matrix. "What will follow if I do this? What will follow if I do not do this? In this way a prudent man should first consider the alternatives and their consequences and then decide to do or not to do a particular act."¹⁰ This is the piece of advice given by a poet to the king Bhoja. It enjoins to take a stock of the consequences or to look before leap. Thus every decision is preceded by deliberation. But as Knight says "decision process involves a lot of irrelevant mental rambling..... it is not reasoned knowledge, but judgment, common sense or intuition. There is doubtless some analysis of a crude type involved but in the main it seems we infer largely from our experience of the past as a whole."¹¹ Marshal holds that decisions are guided by trained instincts rather than knowledge.¹² So most of the real decisions of life are based on the reasoning of a tenuous and uncertain character. They are, more or less, intuitive judgments or unconscious inductions. One of the striking features of these judgments is their liability to error. The logic of decision which is the real logic of ordinary conduct plays here an important role : it tries to minimise the errors and the uncertainties that are likely to beset decisions. It will be a fruitful study to investigate into the various fallacies that vitiate preferability judgments.

Furthermore, it investigates into the calculus or matrix of utility which is the power of things to satisfy conscious wants. The formal logicians have designed desirability functions and related it to belief functions. In this way they have developed a theory of probability judgment which is different from *a priori* or mathematical probability judgment on the one hand and from empirical or statistical probability judgment on the other. All the same this logic of practical reason is still a neglected branch of logic and the efforts are being made to reduce it to either of the probability logics. But conduct is forward looking and the basic problem of knowledge is prediction. So decisions need a new logic that extends the frontiers of formal logic to the

domain of conduct. This logic, as we have discussed above, is not an applied logic, for it has its own logical relations and principles of inference which are lacking in the domain of formal logic. Predictive inferences, quasi-entailments, expectations and likely conclusions are some of the new types of inferences. Similarly order relations like transitivity and symmetry have new meanings and problems in decision situations. All of them are peculiar to the logic of decision. This logic is nearer to Indian conception of logic according to which the removal of uncertainty is the main aim of logic.¹³ Further the principle of insufficient reason tries to sort out knowledge from ignorance and thereby make human knowledge as far as possible reliable and predictive. Whether we should call this knowledge as true or certain or probable, or reliable, or simply as plausible is a point of further debate. For ordinary decision-makers they have, by and large, the same connotation. But the logicians do make a distinction among them and maintain that the logic of decision has more concern with the plausibility-values than any other truth-values. In this context it is significant to note that the ideal of the logic of decision is neither truth nor certainty nor probability.

VI

An example of reasoning in decision : To give an example of reasoning involved in decision I take prisoner's dilemma. Two men are arrested for an armed robbery. The police are convinced that both are guilty but they lack sufficient evidence to convict either. So they put the following proposition to them and then separate them :—

If one man confesses but the other does not the first will go free and the other will get the maximum sentence for 10 years. If both confess, they will both get light sentences for 5 years. If neither confesses, they will both be imprisoned for vagrancy with a total sentence of 1 year each.

What will each prisoner decide ? Let us take the first prisoner. What will he decide ? The Police are convinced that both prisoners will confess, although it would be better for both if neither confessed. Why are the police so convinced ? Here is

the argument : Let P_1 and P_2 be the first prisoner and the second prisoner respectively. The alternatives for p_1 and p_2 are :—

	P_1	P_2	Confess
Not Confess		Not Confess	(10, 0)
		(1, 1)	
Confess		(0, 10)	(5, 5)

The probability matrix for p_1 is as follows :

Not confess	.25	.25
Confess	.25	.25

Taking acquittal as most desirable, i. e., as 1 and conviction for 10 years' imprisonment as — 1 and one year's sentence as — .1 and 5 years' sentence as — .5 the desirability matrix for P_1 is as follows :

Not confess	— .1	— 1
Confess	1	— .5

Now expected desirability on not-confession

$$= (-.25) \times (-.1) + (.25 \times 1) \text{ by multiplying and adding the entries in the first row of the two matrices.}$$

$$= -.025 - .25 = -.275$$

And expected desirability on Confessing

$$= (.25 \times 1) + (.25 \times -.5) \text{ By multiplying and adding the entries, in the second row.}$$

$$= +.25 - .125$$

$$= +.125.$$

Now according to Bayesian principle the maximum expected value is to be selected. Obviously $+.125$ is more than $-.275$. Therefore confession is preferred to non-confession. Hence the first prisoner will confess. Similarly it can be proved that the second prisoner will also confess. So the police are rightly convinced of the fact that both prisoners will confess.

Now the reasoning that has been used above can be abstracted as follows :

Probability matrix can be taken as

P_1	P_2
P'_1	P'_2

Desirability matrix can be taken as

$$\begin{array}{cc} d_1 & d_2 \\ d'_1 & d'_2 \end{array}$$

The multiplication of the two matrices is

$$\begin{array}{cc} P_1d_1 & P_2d_2 \\ P'_1d'_1 & P'_2d'_2 \end{array}$$

By addition of the first row we get

$P_1d_1 + P_2d_2$ which is $- \cdot 275$ in the above case.

By addition of the second row we get

$P'_1d'_1 + P'_2d'_2$ which is $+ \cdot 125$ in the above case.

The rationale of this calculation is the familiar practice in the game of gambling that associates probability with monetary gain or loss. The prisoner's dilemma indicates that the best decision could not be taken by the two prisoners simply because there was no co-operation between them or because the one was totally ignorant or uncertain of what the other was really going to decide. Had they co-operated with each other they would have preferred not to confess. This shows that group decision in such situations is more rational than individual decision. But individuals often take decision under uncertainty, so the prisoner's dilemma fully illustrates the reasoning that is involved in solving the decision problem under uncertainty. It is an example of two person non-zero sum non-co-operative game. What the prisoners decided was the most satisfactory, though not the best, for them under the conditions of uncertainty. In this way logical decisions may not be the best but are the most satisfactory ones under the prevailing conditions of uncertainty. So the logic of decision is not concerned with the best decision; its concern is only the most satisfactory decision which is available in any given situation and the most satisfactory decision may be regarded as the most rational decision.

VII

The reasoning that is involved in decision-making may be termed in a majority of cases, as nomic inference. There are many current rules of conduct in every society. People generally seek guidance from these rules which have been established by inductive reasonings of several generations. They apply them to their situation and get a satisfactory conclusion. Take for example the

prisoner's dilemma. Here a prisoner may think in the following manner:-

A man who seeks the uncertain and leaves the certain, actually spoils both.¹⁴ Therefore what is certain, should be preferred to what is uncertain. Now if I choose the certain, there are two positions. First if I confess, at best I shall be acquitted or at worst I will get a sentence of five years. And if I do not confess at best I shall get a sentence of one year or at worst I shall get a sentence of 10 years. When I choose between the best situations, I have to confess and if I choose between the worst situations, again I have to confess. Therefore I must confess.

Here the prisoner makes an analysis of his situations which are best and worst. Then he makes a comparison between the best situations on the one hand and between the worst situations on the other hand and finds that in either circumstances he must confess. Now the danger in confessing is only to get a sentence of 5 years. But the danger in not confessing is to get a sentence of 10 years. So a greater evil is to be avoided by choosing a lesser one. This conclusion is also confirmed by the rule that the prudent men give up the half where there is a possibility to lose the whole or by the rule that the golden mean should be preferred.

The above reasoning has been elegantly presented in the logic of decision in terms of probability function and desirability function. But when it comes to be represented in the ordinary language of common people, it is based, as we have shown, on certain rules of conduct. These rules may not be universally applicable or valid. But they are valid to some types of relevant human situations and this is the reason why they are proverbially used and propagated by the common genius of the people. A prudent man must learn as many of them as possible, for this learning yields the most important set of instrumentalities or tools for decision-making. But it should not be forgotten that these tools are only the rules of thumb and that they cannot take the place of the logic of decision which deals with a set of sophisticated rules for decision-making and develops the criteria for testing them. These rules of thumb, however, indicate an important feature of the logic of decision that its reasoning is, by and large, nomic inference which may be taken mid-way between deductive logic and inductive logic as it makes inferences from quasi-universal rules to particular propo-

sitions or relevant consequences. But the rules of thumb themselves require a logical treatment for their consistent exposition and the analysis of the principle that are hidden behind them.

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NOTES

1. *Games and Decisions*, R. Luce and H. Raiffa, New York, 1957 pp. 286-298.
2. *The Logic of Decision*, R. E. Jefferey, McGraw Hill Book Company, New York, 1965, p. 199.
3. *Ibid.*, p. 199.
4. *Local Induction* ed. R. J. Bogdan, D. Reidal Publishing Company Dordrecht, Holland, 1976, pp. 93-94.
5. *The Coherence Theory of Truth*, Nicholas Rescher, Oxford, 1973, pp. 98-99.
6. *Ibid.* p. 28.
7. See for the concept of decisionary effort, *Decision Theory*, D. J. White, London,
8. *Expectation, Enterprize and Profit*, G. L. S. Shackle, London, 1976, p. 106.
9. See for the classification of decision theory, *Encyclopaedia of Philosophy* ed. Paul Edwards, Vol. p. 310. A workable classification of decision making is given on *Games and Decisions*, R. Luce and H. Raiffa, New York, 1957, p. 13.
10. Kim nu me syād idam Krtvā, kim nu me syād akurvātā. Iti sañcintya manasā pranāṇ kurvita vā na vā. *Bhoja Prabandha*, 23.
11. *Risk, Uncertainty and profit* : F. H. Knight, New York, 1946, p. 211.
12. Quoted Op. Cit. p. 211.
13. atra nānupalabdhe na nirṇīte arthe nyāyaḥ pravartate kim tarti? sañśayite arthe. *Vātsyāyana's* commentary on *Nyāyasūtras* 1.1.1.
14. Yo dhruvāṇi parityajya adhravaṁ parisevate. Dhruvāṇi tasya naśyanti adhravaṁ naśtameva ca. Quoted in *Subhāṣitaratnabhāṇḍāgāraṁ* ed. Narayana Rama Acarya, Nirnaya Sagar press, Bombay, 1952, p. 162.

