IS FARIS' DERIVATION OF 'IF P THEN Q' FROM 'P⊃Q' TENABLE?

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This paper refers to "'If... then' and Horse Shoe ('⊃')- A Strawsonian Account" of Kantilal Das published in this Journal.[1] In this paper the author tries to show the interderivability of 'if... then' and '⊃'. There is no question of disagreement among logicians about the derivability of 'p⊃q' from 'if p then q'; but all differences come off from the derivability of 'if p then q' from 'p⊃q'. Before starting our argument for the derivability of 'if p then q', from 'p⊃q', let us first reflect on the notion of 'if... then' (*ordinary implication*) and '⊃' (*material implication*).

A statement composed of two constituent statements and the phrase 'if... then' is called a conditional. The component statement that precedes 'then' is called the *antecedent* (meaning 'that which precedes'), and the component following 'then' is termed the *consequent* ('that which follows'). The key to the meaning of a conditional is the relation of implication that is asserted to hold between its antecedent and consequent, in that order. In the conditional *'If all men are mortal and Plato is a man, then Plato is mortal'*, the consequent logically follows from its antecedent, and hence the relation between the antecedent and the consequent is logical. In the conditional *'if the figure is a triangle, then the figure has three sides'*, the consequent follows from the antecedent by the very definition of the word 'triangle', and hence the relation between them is a definitional one. In the conditional *'if blue litmus paper is placed in acid, then it will turn red'*, the consequent neither follows logically from the antecedent nor does it follow from the very definition of any word contained in the antecedent, but the connection asserted here is causal, and hence it must be discovered empirically.

If we examine a number of different conditionals, then we can see that there are different kinds of implications that constitute different senses of the 'if... then' phrase used in ordinary language. Hence they are called *ordinary implications or relational implications.* In these conditionals the relation between the antecedent and the consequent is so intense that the truth or falsehood of the consequent cannot be determined without taking recourse to the truth or falsehood of the antecedent. For example, in the conditional "*If we take medicine, then our disease will be cured,*" if 'we take medicine' and 'our disease is cured' are true, then we will consider the conditional to be true. But if 'we take medicine' is false, that means, if we do not take medicine, then our disease may or may not be cured, and hence nothing can be said about the conditional. Sometimes it happens that if 'our disease is cured' is true, then 'our taking medicine' may be true or false, and hence nothing can be said about the conditional. Again if 'our disease is cured' is false, that means if our disease is not cured, then 'our taking medicine' may be true or false, and hence nothing can be said about the conditional. Supposing 'p' for the antecedent and 'q' for the consequent, we have the following results concerning *ordinary implication :*

(I) If 'p' is true and 'q' is true, then the implication is true.

(II) If 'p' is false, then 'q' may be true or false, and hence the implication remains undetermined.

(III) If 'q' is true, then 'p' may be true or false, and hence the implication remains undetermined.

(IV) If 'q' is false, then 'p' may be true or false, and hence the implication remains undetermined.

There is another type of conditional with no connection between its antecedent and consequent. It is special and non-relational in nature. In the conditional '*If* 2+2 =2, *then London is a small city*', there is no connection between its antecedent and consequent. This type of conditional is called material implication or Russellian implication. The notion of *material implication occurs in Principles of Mathematics.* It is truth functional in character. The truth value of this implication is determined by means of a mechanical process. Any statement of the form 'p⊃q' ('p' materially implies 'q') is true if "it is not the case that the first of its constituent statements is true and the second false, and false if and only if the first of its constituent statements is true and the second false. That is to say, the falsity of the first constituent or the truth of the second is equally a sufficient condition of the truth of a statement of material implication and the combination of the truth in the first with the falsity in the second is the single necessary and sufficient condition of its falsity".[2] Thus we have the following results concerning *material implication :*

I) If $v(p) = T$ ***and*** $v(q) = T$, ***then*** $v(p \supset q) = T$

II) If $v(p) = T$ ***and*** $v(q) = F$, ***then*** $v(p \supset q) = F$

III) If $v(p) = F$ ***and*** $v(q) = T$, ***then*** $v(p \supset q) = T$

IV) If $v(p) = F$ ***and*** $v(q) = F$, ***then*** $v(p \supset q) = T$

J. A. Faris, in the last part of his monograph 'Truth Functional Logic'[3] claims that *ordinary implication* can be derived from *material implication.* That is to say, 'if p then q' can be derived from 'p⊃q'. But my aim of writing this paper is to show whether or not his argument is sustainable. To derive 'if p then q' from 'p⊃q', Faris suggests an essential condition by means of which the necessary condition of the truth of 'if p then q' could be established. That condition is the condition-E, which runs as follows:

"There is a set S of true propositions such that 'q' is derivable from 'p' together with S".[4] This condition is a sufficient and a necessary condition of the truth of 'if p then q'. The derivability of 'if p then q' is possible with the aid of condition-E. According to Faris 'q' can certainly be inferred from 'p' and 'p⊃q'. Accordingly if 'p⊃q' is true, then there is a set S of true propositions, namely the set consisting solely of the proposition 'p⊃q', such that 'q' is inferrible from 'p' together with S. It follows that if 'p⊃q' is true, then condition-E is satisfied. But if condition-E is satisfied, then 'if p then q' is true. Consequently if we are able to know that the proposition 'p⊃q' is true, then we are able to know that the proposition 'if p then q' is true, also whatever other truths or falsehood there may be. That is to say, 'if p then q' is derivable from 'p⊃q'.[5]

Prof. Baker[6] criticises Faris' arguments in maintaining the view that 'if p then q' cannot be derived from 'p⊃q', because 'if p then q' is sometimes consistent and inconsistent with 'if p then not- q', while 'p⊃q' is consistent with 'p⊃~q'. If p then q' and 'if p then not-q' are consistent when they occur as premises of a *simple destructive dilemma. By simple destructive dilemma* we mean, a dilemma whose minor premise alternatively denies the consequents of the compound hypothetical major premise and the conclusion is a categorical proposition. Thus 'if p then q' and 'if p then not-q' are consistent as in such an argument as "if we are to escape, we must retreat (because the enemy is advancing), and if we are to escape we must not retreat (because there is a precipice behind us), therefore we shall not escape.[7] 'If p then q' and 'if p then not-q' are inconsistent is case of *ordinary hypothetical,* as "if the meeting is held at night, most members will attend and if the meeting is held at night most members will not attend".[8] It is plain that these two are inconsistent statements, only one of which can be true.

On the otherhand any two *material-implication-statements* of the form 'p⊃q' and 'p⊃~q' are consistent with one another. Because in *material implication* the falsity of the antecedent is a sufficient condition of its being true. For example, to say, 'it will not rain, the falsity of the antecedent of "it will rain ⊃ the match will be cancelled", is to say that this condition is suficient to establish the truth of this *material-implication-statement.* Thus when 'p⊃q' and 'p⊃~q' are both true (consistent), Faris' derivation of 'if p then q' from 'p⊃q' is not possible. Because according to condition-E, whenever 'p⊃q' is true, condition-E is satisfied. If condition-E is satisfied, then 'if p then q' is derivable from 'p⊃q'. Similarly, whenever 'p⊃~q' is true, condition-E is also satisfied. If condition-E is satisfied, then 'if p then not-q' is derivable from 'p⊃~q'. Thus according to condition-E, whenever 'p⊃q' and 'p⊃~q' are both true, we would be able to derive the corresponding hypotheticals, 'if p then q' and 'if p then not-q', and thus commit to the truth of both of inconsistent propositions. But to derive 'if p then q' from 'p⊃q' by means of condition-E, not only 'p⊃q' be true but also 'p' is true. In that case we would not be able to derive 'if p then not-q', which is inconsistent with 'if p then q', for.'p⊃~q' would then be false.

Russell[9] does not accept Baker's argument, but suggests that 'if p then q' and 'if p then not-q' are *always* consistent, otherwise *reductio-ad-absurdum* (R.A.A.) argument would not be possible. But 'if p then q' and 'if p then not-q' might be inconsistent if and only if an additional assumption is made. that is to say, given the truth of 'p', 'if p then q' and 'if p then not-q' would be inconsistent. Thus it is to be noted that for R.A.A. to be possible 'if p then q' and 'if p then not-q' must be consistent. Because if 'if p then q' and if p then not-q' are both true, then the conclusion, '–p' could validly be drawn. But this argument of Russell is not sustainable for the following reasons.

If 'if ... then' is understood in terms of *material implication,* then the possible falsity of the antecedent would be a decisive one. Thus the formula 'p⊃q' is consistent with 'p⊃–q' exactly because 'p' may be false, i.e.

1.	p ⊃ q	p ⊃ ~ q
	F T T	F T F T
2.	p ⊃ q	p ⊃ ~ q
	F T F	F T T F

But if 'if... then' is understood in terms of *ordinary implication,* then the possible falsity of the antecedent would not be a sufficient condition of its being true. For example, 'it will not rain' or 'it does not rain' is not considered as a sufficient condition of the truth of such an *ordinary implication* as 'if it rains, then the match will be cancelled'. The *ordinary implication* is taken to be true when both the antecedent and the consequent are found to be true. Thus "if it rains, the match will be cancelled" is taken to be inconsistent with 'if it rains, then the match will not be cancelled', and hence their joint assertion in the same context is self-contradictory. But the *material-implication*-sentence, "it will rain ⊃ the match will be cancelled" is surely taken to be consistent with the corresponding *material-implication* sentence, "it will rain ⊃ the match will not be cancelled". Thus the formulas 'p⊃q' and 'p⊃~q' are consistent with one another and their joint assertion is equivalent to'–q'. This could be demonstrated as :

{1}	(1) (p⊃q). (p⊃–q)	/∴–p
{2}	(2) p	
{1}	(3) p⊃q	I, T

{1,2}	(4) q	2, 3, T
{I}	(5) p⊃~q	1 T
{1,2}	(6) ~q	2, 5, T
{1,2}	(7) q, ~q	4, 6, T
{1}	(8) ~p	2, 7, R.A.A.

But if 'if... then' is understood in terms of *strict implication* (*strong Implication*), then 'if p then q' is true when 'q' is deducible from 'p'. Again, If 'if... then' is understood in terms of *relevant implication* then the validity of an inference from 'p' to 'q' is that 'p' be relevant for 'q' . Thus in the proposition 'if p then q' the antecedent 'p' must be true, its actual truth value being disregarded as irrelevant to the truth value of the conditional. The truth of the antecedent may be relevant to the possibility of verifying or falsifying the conditional, but not to its truth value. Everywhere but in truth functional logic the validity of an inference is independent of the truth value of the antecedent or the premises. In truth functional logic, however, the truth value of 'p⊃q' depends on the truth value of 'p', if 'q' is false. But because we have to disregard as irrelevant the possible falsity of the antecedent, since it is supposed to be true, the propositions 'if p then q' and 'if p then not-q' are inconsistent, whereas 'p⊃q' and 'p⊃~q' are not. Thus it is to be noted that the consistency / inconsistency between 'if p then q' and 'if p then not-q' depends on the kind of logic we take for interpretation.

But 'p⊃q' and 'p⊃~q' could quite well be inconsistent if and only if a tautology is substituted for 'p'. That is to say, '(p⊃q) ⊃q' and '(p⊃q) ⊃~q' are inconsistent, because they cannot both be true. This is proved by means of truth-table.

1.	(p⊃p) ⊃q	(2)	(p⊃p) ⊃~q
	T T T T T		T T T F F T
	T T T F F		T T T T T F
	F T F T T		F T F F F T
	F T F F F		F T F T T F

Thus Russell's thesis, "if p then q' and 'if p then not-q' are *always* consistent"[10] is not tenable at all.

As Russell states, "for a *reductio-ad-absurdum* argument to be possible, 'if p then q' and 'if p then not-q' should be consistent".[11] By *reductio-ad-absurdum* we mean : "if a contradiction is derivable from a set of premises and the negation of the formula S, then S is derivable from the set of premises alone, i.e. $P \cup -s \vdash (s.-s) \supset P \vdash S$".[12] That means, if we derive a contradiction from a set of premises, then it is proved that at least one of the premises in that set is false. This follows from the fact that we cannot derive false sentences from true ones, plus the fact that contradictions are false sentences. Now, consider, say, a four premise argument, and assume that three of its four premises are true. Then, if we derive a contradiction from that set of four premises, then it is proved that the fourth premise in that set is false (since at least one member of the set is false, and we assume that the other three are true). But it follows then that we also have proved that the negation of the fourth premise is true (since the negation of a false sentence is true). We have here the main idea behind the *Indirect or reductio-ad-absurdum* (R.A.A.) proof. This could be demonstrated as :

{1}	(1) p ⊃ (q.r)	
{2}	(2) (q v s) ⊃ t	
{3}	(3) (s v p)	/∴t
{4}	(4) –t	(1.p)
{2,4}	(5) – (q v s)	2, 4 T
{2,4}	(6) –q. –s	5 T
{2,4}	(7) –q	6 T
{2,4}	(8) –s	6 T
{2,3,4}	(9) p	3, 8 T
{1,2,3,4}	(10) (q . r)	1, 9 T
{1,2,3,4}	(11) q	10 T
{1,2,3,4}	(12) q . – q	7, 11, T
{1,2,3}	(13) t	4, 12.R. A. A.

But the argument of Russell, which really exemplifies the formula '((p->q). (p->–q)) - > –p', (where '->' is equivalent to 'if... then), also fails. Because such an argument arises when 'q.~q' is substituted for' 'p', the result being '(((q.~q). ((q.~q)->~q)) ->~(q.~q).' It shows that "~(q.~q) is derived from apparently avlid conditionals, '(q.~q)->q' and '(q.~q)->~q', not because they are consistent, but on the contrary, because they are inconsistent. From inconsistent prositions it may be inferred that their premises or antecedents can not all be true, and here the only antecedent in question is 'q.~q'. Thus it is precisely because 'p->q' and 'p->~q' (if p then q' and 'if p then not-q') are inconsistent, that they imply ~p".[13]

Now we can paraphrase the argument as follows :

(i) 'p⊃q' and 'p⊃~q' are consistent.

(ii) 'If p then q' and 'if p then not-q' are inconsistent.

(iii) When 'p⊃q' and 'p⊃~q' are consistent, we would be able to derive their corresponding inconsistent hypotheticals, 'if p then q' and 'if p then not-q'

(iv) When 'if p then q' is derived from 'p⊃q', at that time not only 'p⊃q' be true, but also 'p' is true. In that case we would not be able to derive 'if p then not-q'.

Note that Russell, in his paper "if and ⊃", *Mind*, 1970, V-LXXIX, p. 135, makes no distinction between 'if... then' and 'hook', and claims that the relation between 'p⊃q' and 'if p then q' are not peculiar to the 'hook'. So 'if p then q' is equivalent to 'p⊃q'.

It is, therefore, concluded that Faris' derivation of 'if p then q' from 'p⊃q' is not tenable.

NOTES

1. Das, K : "If... then' and Horse shoe ('⊃') - A Strawsonian Account", *Indian Philosophical Quarterly,* 1997, V-XXIV, N-1, pp. 41-51
2. *Ibid.* p. 42
3. Faris, J. A : *Truth-Functional Logic,* London, Routledge & Kegan Paul, 1962, p. 109.'
4. *Ibid.* p. 117

5. *Ibid.* p. 118

6. Baker, A. J. 'If and '⊃' *Mind*, 1967, V-LXXVI, N-303, p. 437

7. *Ibid.* p. 438

8. *Ibid*

9. Russell, L. J. : 'If and '⊃' *Mind* 1970, V-LXXIV, N-313, pp. 135-36

10. *Ibid.* p.136

11. *Ibid.* p. 135

12. Suppes, P : *Introduction to Logic,* Affiliated East-West Press Pvt. Ltd., New Delhi, 1978, p. 40

13. Norreklit, L : 'On If and '⊃' *Mind* 1973, V-82, N-325, p. 443