# PUTNAM'S PROPOSAL FOR LOGICAL REFORM

N. G. KULKARNI

In this brief paper I shall consider a specific proposal by Putnam to replace classical logic by quantum logic. There are many puzzles and controversies in this highly esoteric branch of mathematical physics. There is the debate concerning deterministic vs. statistical laws. There are the mysteries concerning the behaviour of the electron as revealed, for example, by the phenomenon of 'superposition' which suggests that an electron has passed through both of two apertures when the experimental set-up makes it possible for it to pass through only one of them. Finally there is the principle of indeterminacy. It is impossible to determine, by the most sophisticated methods, both the position and the momentum of an electron with precision. The more accurate the value of one of the variables the less accurate correspondingly, is the value of the other. The product of the two errors is constant and this suggests that the imprecision is not due to practical limitations but is somehow, inherent in the situation.

Suppose $p_1$ is the ascertained position of an electron and $m_1$, v $m_2$, are its possible momenta. The very fact of ascertaining the position makes it impossible (theoretically) which one of the possible momenta, the electron has. It is therefore true in Quantum Mechanics (using '$p_1$' and '$m_1$ v $m_2$' symbolise the statements concerning the precise position and alternative momenta of the electron) that '$p_1$' and also that '$m_1$, v $m_2$' but it is not ascertainable either that '$p_1.m_1$' or that '$p_1.m_2$'. Thus the distributive law has to be jettisoned. The difficulty can be tackled by advancing scientific theories of a highly general and speculative character; the principle of complementarity is one of them.

Putnam, however, prefers to remove the difficulties by adopting an

*Indian Philosophical Quarterly* XXIX No 2 & 3
April- July 2002

alternative to classical logic. He explicitly compares this move to the adoption of elliptical geometry, in place of Euclidean, by Einstein to deal with light rays passing through powerful gravitational fields-such as the sun's. Putnam's proposal is not the first proposal of its kind nor is it the first attempt at reforming logic. Way back in the thirties Neumann and Birkhoff proposed a logic without the principle of excluded middle. Their logic was not 3-valued nor was it truth functional. When Lukasiewicz expounded, fairly systematically, his 3-valued logic, Reichenbach proposed the adoption of such a logic for the purpose of dealing with microcosmic phenomena. Putnam himself, in an earlier essay on three-valued logic, approved of this suggestion. Later on, however, he came to feel that this would not accomplish the desired result. In an essay entitled "Is Logic Empirical?", first published in 1968 and reprinted in the 1st volume of his '*Philosophical Papers*' under the title "The Logic of quantum mechanics" he argued for his proposal to reject the distributive principle at some length.

He assumes that scientific theories including quantum mechanical theoris are to be understood "realistically". That is to say, they are not to be understood in purely operational or instrumental terms. Nor are we to have recourse to purely *ad hoc* hypotheses or metaphysical assumptions concerning the relation between the observer and the observed. In the above symbolical example the electron has some particular momentum, though we cannot specify whether it is $m_1$ or $m_2$. Neither the momentum nor the position of the electron is brought into being by our methods of measurement. So it is true that $m_1 \vee m_2$ just as much as it is true that $p_1$ But it is not true either that $p_1 . m_1$ or that $p_1 . m_2$. Why does Putnam deny the truth of these conjunctions instead of their knowability? The reason seems to be that having ascertained the truth of $p_1$, I cannot ascertain the truth of either $m_1$ or $m_2$ so that if I knew either $p_1.m_1$ or $p_1 . m_2$ I would know a logical contradiction. Let us grant that the impossibility of knowing both the position and the momentum of an electron is a theoretical impossibility and not merely the result of our ignorance. This theoretical impossibility can, in a loose sense, be termed logical. But surely, even if we equate truth with knowability-which can be done with some necessary qualifications-it does not follow that any one who knew either of the two conjunctions would be knowing a logical contradiction. Knowing what is false is impossible

conceptually, and since a logical contradiction is the paradigm of falsity, it is logically impossible to know a contradiction. But every impossibility of knowledge does not reduce to knowledge of impossibility, so that we cannot equate "impossible to know that p" with "knowing P which is impossible". Since the realism adopted by Putnam implies that $m_1$ v $m_2$ is not only meaningful but true, and $p_1$ is known to be true, the conjunctions '$p_1 m_1$' and $p_2.m_2$' can *neither of them be regarded as logically contradictory* though knowing either of them can be regarded as logically contradictory, in some sense. Putnam thus seems to be mixing up truth-values with noetic or epistemic values. Once this is realized it can be shown that there is no need to discard the distributive principle to avoid a contradiction in Quantum Mechanics. Let us use 'K p' to symbolize "it is known that p". Then the situation in "Quantum Mechanics is summarized by the following three formulae :

1. $Kp_1$
2. $K (m_1 \text{ v } m_2)$
3. $\sim K (p_1.m_1). \sim K (p_1.m_2)$

A possible derivation of a contradiction could run as follows.

4. $Kp_1\text{-}K(m_1 \text{v} m_2)$.....1, 2 Conj.
5. $K [p_1.(m_1 \text{ v } m_2)]$4, by acceptable thesis of modal logic.*
6. $K[p_1.m_1) \text{ v } (p_1.m_2)]$.....4, distri.
7. $K (p_1.m_1) \text{ v } K(p_1.m_2)$.....6 dist. of K Sign. over disjuncts (?)
8. $\sim[\sim K(p_1.m_1) .\sim K (p_1.m_2)]$.....7 De Morgan's

* We treat the K-sign as formally analogous to the sign for logical necessity.

Step 8 contradicts the premiss 3.

The sixth step in this attempted derivation is clearly fallacious. Since the 'K' sign is farmally analogous to the necessity sign in modal logic and the universal quantifier in elementary predicate calculus. We can bring home the fallaciousness of the above step by noticing that

1. $(x) (Fx \text{v} Gx) = [(x) Fx \text{ v } (x) Gx]$
2. $N (p \text{v} q) = (Np \text{ v } Nq)$

are obviously fallacious though the converse implications are valid.

On the other hand, if we insist that '$p_1$' and "$m_1 \vee m_2$" are both true and '$p_1.m_1$' and '$p_1.m_2$' are both false in quantum logic, though not in classical logic, It can be shown, I think, that a contradiction results. Putnam, through out his discussion, gives the impression that it is sufficient to sacrifice the distributive principle to remove anomalies from Quantum Mechanics. At any rate he does not mention any other principle that has to be jettisoned. On the other hand he insists that we can retain a number of laws of classical logic including the principles of excluded middle, double negaion, simplification, conjunction and addition. He also adds P.~P never holds even in quantum logic. We may set up the following simple derivation.

| | |
|---|---|
| 1. $p_1$ | Premiss |
| 2. $m_1 \vee m_2$ | Premiss |
| 3. $\sim (p_1.m_1) . \sim (p_1.m_2)$ | Premiss |
| 4. $\sim (p_1.m_1)$ | 3 simpli |
| 5. $\sim p_1 \vee \sim m_1$ | 4 De Morgan |
| 6. $\sim \sim p$ | 1 D.N. |
| 7. $\sim m_1$ | 5, 6, Disj Syll. |

By similar steps, from the second conjunct in 3 we obtain

| | |
|---|---|
| 8. $\sim m_2$ | |
| 9. $\sim m_1.\sim m_2$ | 7, 8, Conj. |
| 10. $\sim (m_1 \vee m_2)$ | 9 De Morgan's |

Step 10 contradicts premiss 2

The only principles used in the derivation, not explicitly accepted by Putnam in his writings are, De Morgan's law and the principle of the disjunctive syllogism. The former is a thesis in the 3 valued logic of Lukasiewicz as also in Heyting's system of intuitionist Logic. The principle of the disjunctive syllogism is not a law of Lukasiewicz's system. I do not know what Putnam's attitude would be on admitting them into his version of quantum logic. The princple of D. N. is a law in Lukasiewicz's system and I have used that half of the equivalence which is acceptable to Brouwer. He will have to reject at least one of them. And it is likely that once this is

done other revisions may be required. The cost of reforming logic may be heavier than Putnam imagines.

In the essay referred to above, as also in "Two Dogmas revisited", Putnam insists that alternative logics are on par with alternative geometries. And yet there is a crucial difference. On a physical interpretation the three geometries result in incompatible theorems. Neither intuitionist logic nor the 3-valued Logic of Lukasiewicz contains a thesis incompatible with any of the principles of classical logic. So far as I know, no attempt has been made to develop such a logic. So far as the 3-valued logic is concerned the best description of the situation is to say that under the rules governing logical constants, extended to cover the intermediate value, some of the principles of classical logic cease to be tautologies. But then there is ample reason to belive that. The numerical values of the system represent, not truth values but certainly values or noetic values. Moreover, if logic can be changed and chopped to suit a particular inquiry, there is no reason why different logics should not be employed in different fields. Logical principles will then exhibit local or geographical variations. Leaping electrons may defy the distributive law but an ambitious father who wants for his daughter, a lier who is either a well-placed civil servant or a green card holder, will be perfectly content with some young man who is both a lier and a well placed civil servant or else both a lier and a green card holder. And rightly so; satisfaction of either of the two formulations will fullfil his heart's desire and neither will give him any thing more than the other.