

## BOOK REVIEW -II

Katz, Jerrold J. *REALISTIC RATIOALISM*, The M I T Press (A Bradford Book, Cambridge, Mass., 1998, pp.XXXIV + 226, Price : US \$ 47. 50 )

It has been usually thought that Rationalism leads to Idealism. In modern times the origins of idealism have been traced to Descartes' *Cogito*. The career of idealism has passed through solipsism, subjective idealism and pluralistic or monistic forms of idealism with all sorts of shades and shines. Rationalistic Idealism has been always opposed by the school of Realism on empirical grounds. The dispute between idealism and realism has been age-old and the rivalry between the two was considered to be unending. A wide-ranging conviction holds in the present philosophical world that the real and the rational cannot go together. The linguistic turn in philosophy during the 20th century, no doubt, brought some relief in this controversy by tending to show that if we resort to the methods of logical and linguistic analysis, the controversy itself could be shown to be frivolous and hence unnecessary. But this respite, as every one in the field knows, is proved only shortlived and temporary. We have fast begun to realize that the philosophical problems cannot be dissolved and like the soul in Bhagvatgītā, they go on wearing new attires.

The book under review is a serious attempt towards showing that realism and rationalism can be integrated. This is indeed a new philosophical position, quite unlike Plato's or Frege's, and Jerrold J. Katz has arrived at it after almost a career-long endeavour. Katz has certainly emerged as an influential thinker of this decade, in the field of philosophy of language. He has kept himself on the front against the naturalistic strong winds of the Wittgensteinian and Quinean sail-boats. In his

*The Metaphysics of Meaning* ( The MIT Press, Cambridge, 1990), he had offered a radical reappraisal of the "linguistic turn" in 20th century philosophy and showed that the naturalism that emerged from such a turn to become the dominant philosophical position was never adequately proved. In that book he had criticised in detail the arguments of Wittgenstein, Kripke and Quine supporting naturalism of one sort or another and had developed, taking cue from Moore, a new conception of the naturalistic fallacy. By employing that conception, he had explained why attempt to naturalize linguistics and logic will always fail. Offering a platonistic view of such disciplines, Katz had justified it as the best explanation of their autonomy, objectivity and normativity. The present book, viz., *Realistic Rationalism* is a further reassessment and the author claims:

"It is a radical reassessment. Its broad aim is to provide the metaphilosophy and the arguments to show that abandoning the traditional conception of philosophy in favour of one or another form of naturalism was a fundamental mistake. Not that traditional versions of the metaphysical conception of philosophy did not deserve criticism, but the critics threw out the baby with the bathwater. .... The present book takes the next step of formulating and justifying a new version of traditional realist and rationalist philosophy." (p. xvi-xvii).

The position which Katz takes in this book is the result of his thorough disillusionment with naturalism which was the outcome of finding that "naturalist and empiricist philosophies do not provide satisfying answers to the questions that first lure us into philosophy.....and of coming to think that answering some of those questions requires a non-naturalist position combining realism in ontology with rationalism in epistemology" (p. xvii).

The Introduction to the book presents comments on the first-order-second-order distinction of disciplines and on attempts to treat philosophy exclusively as a second-order discipline with the ensuing consequence that there can't be an autonomous metaphysical philosophy. Katz is opposed to this kind of hard and fast distinction and suggests that his non-naturalis-

tic position claims that philosophy is both first-order and second-order discipline "It is first order discipline only in asking questions about the world. "The traditional philosophers who took philosophy to be an inquiry into general facts about reality did not, I think, want to say that philosophy is part of the scientific enterprise in a hands-on way." ( p. xviii ). One should not ignore that in the course of scientific investigation of reality philosophical questions do arise. They concern the nature and validity of the very methodology on which scientific knowledge rests. Problem of induction e.g. is a philosophical issue and one can see its intimate relationship with skepticism in science which is an another philosophical issue. It would be incorrect to suppose that philosophical issues in the field of science are faced by philosophers alone and not by the scientists. There are philosopher-scientists who take philosophical problems in their field seriously enough. Katz maintains that on his position, "philosophy, conceived of as a second order discipline.....nonetheless answers certain questions about the objects in the domain of sciences. "(p.xix) How does it go about doing this? This is the major question to which the entire book is devoted as an answer. According to Katz, the philosophy of mathematics which is mainly concerned with foundations of mathematics would be a paradigm example to provide this answer which can then be extended to the field of physical science. Rest of the Introduction is mainly intended to indicate in brief the major points of difference which he holds with the naturalistic and empiricistic positions held from Frege onwards. It contains reflections on positivistic, Wittgensteinian, and Quinean positions with a view to explaining how Katz himself moved from the position of a scientific naturalism to Realistic Rationalism which he is defending in the book. It's a sort of charting out his own intellectual development in his career as a linguist. It tells us how he moved away from Quine and Chomsky to mark the first break with naturalism in his book *Language and Other Abstract Objects* (1981). In that book, Katz had argued that:

"linguistics is a science of languages, collections of sentences - not minds - and its theories are about abstract objects in the same sense in which mathematical realists claim that mathematical theories are about abstract objects" (p.xxvi)

He came to develop side by side a non-Fregean and non-Carnapian definition of sense. Katz holds the view that "sense is the determiner of sense properties and relations, like meaningfulness and synonymy, rather than the determiner of referential properties and relations, like denotation and truth." (p.xxvi) He also maintained that this non-Fregean intensionalism of his is free from the difficulties attributed to Fregean intensionalism. The new intensionalism suggests the rejection of naturalism based on linguistic realism. In *The Metaphysics of Meaning* (1990) Katz had argued that

"Wittgenstein's case for ontological and epistemological naturalism and Quine's case for his uncompromising empiricism and methodological naturalism were based on weaknesses in the Fregean foundations for realism."(p.xxviii)

Katz had shown that their criticisms were specially tailored to Fregean intensionalism drawing attention to its problems and failure to extend realism about logic and mathematics to natural language. These criticisms, Katz had argued, had no force against the radically different intensionalism that is available by locating semantics within the study of natural languages in linguistics and the linguistic realism it implies. *Realistic Rationalism*, the focus of the present review, vindicates realism in the philosophy of mathematics as a fulcrum for Katz's general defense of Ontological realism - a sort of metaphilosophical framework to hound every possible argument in the camp of anti-realists. Katz holds the view that " vindication of mathematical realism leads straightforwardly to the restoration of the traditional metaphysical conception of philosophy".(p.xxix) The success of the book will certainly depend upon the extent to which mathematical realism succeeds and the defence of the metaphilosophical position which he takes. The task of restoring rationalistic conception of philosophy through mathematical realism is a difficult task. The labours which Katz has passed through in accomplishing it are simply amazing. I shall not venture to say at this juncture that Katz has reached a final and conclusive pose to turn tables against naturalism in all its forms but I can vouch this much that the arguments he presents in the book are strong enough to awaken us from the unnerving and demoralizing effect of naturalism, skepticism and deconstructionism. The

Introduction mentions three basic objections in the philosophy of mathematics to the realist claim that mathematics is about abstract objects and author's line of responding to them in brief. Katz claims that "one of the major themes of this book is inseparability of realism and rationalism" and further that "realism without rationalism is unbelievable and rationalism without realism is unstable."(p.xxxii) After the Introduction which, though lengthy, is quite informative and illuminating in developing a proper perspective and clarifying Katz's approach, the whole book is divided into six chapters. The first deals with a few philosophical preliminaries, the second deals with the epistemic challenge to realism, the third with the epistemic challenge to antirealism. The 4th and the 5th, take care of semantic and ontological challenge to realism. The last chapter asks us to move 'Toward a Realistic Rationalism' In the remaining part of this Review, I shall briefly summarise the contents of these chapters, and thereby Katz's line of argument.

The 'Philosophical Preliminaries' which spawn the first chapter speak about i) the framework ii) two forms of antirealism - the Kantian compromise and the Fictionalist nominalism -, iii) Classical Platonism, Contemporary Aristotelianism and Naturalised Realism as wrong turns in the right direction and finally iv) the Moral which a realist can draw from the preliminaries. Starting with the assertion that "Being an object that necessarily has no spatial or temporal location is the core of conception of an abstract object in realist thought from Plato to Godel (p.1), he makes clear that in this book he assumes general realism for the sake of argument and proceeds to defend it against criticisms that we cannot have knowledge of abstract objects, that we cannot determinately refer to them and that we cannot distinguish them from concrete objects. Katz promises us to strengthen his case for realism by supplying reasons to think that abstract objects exist and by exposing weaknesses of one or another form of antirealism. (p.4) He also maintains that

"To establish general realism, it suffices to establish mathematical realism, logical realism or linguistic realism. The argument for establishing one of them is an argument to show that the particular

realism in question is preferable to its rival particular nominalisms and conceptualisms as an account of the objects of knowledge in the relevant formal science. Hence, to explain how a realist can argue systematically for the existence of abstract objects in the domain of a formal science, we have to look at what is involved in showing that realism provides the best account of the objects of knowledge in that science." (pp.4-5)

Distinguishing clearly between the foundations of mathematics, the foundations of logic and the foundations of linguistics as philosophical disciplines from mathematics proper, logic proper and linguistics proper, Katz makes a claim that in virtue of our acceptance of what mathematics, logic and linguistics proper tell us, philosophers are committed to the existence of abstract objects in the related field. The argument that the objects of a formal science are abstract and that they exist is successful if and only if it shows that the realist ontology is best suited to accommodate the full range of facts in any formal science. Katz denies that abstract objects are characterised exclusively on the basis of the negative property of not having spatial or temporal location.

So far as the Kantian view of mathematics is concerned, Katz's observations are indeed penetrating and place finger on the difficulties arising out of his (Kant's) transcendental idealism. Brouwer's mathematical intuitionism is grounded in Kant's old intuitionism but the Copernican revolution which is the turn-key project in Kant's *Critique of Pure Reason*, rests on philosophical doctrines that are too dubious. Katz further mentions two important problems in this context. One is that Kant's transcendental idealism fails to solve the problem about necessity of mathematical statements. Kant locates the grounds of necessity within us - the finite or contingent human beings but, as Frege argued at length, that does not help us explain the necessity of mathematical truth. The other problem is the verificationism in Kant's position which fails to motivate an epistemic constraint on what there is.

Another form of anti-realism stated and examined by Katz is Field's Fictional Nominalism which presents argument against mathematical re-

alism. As against Quine and Putnam, Field attempts to show that natural science can be done without any ontological commitment to abstract objects rendering the indispensability principle of ontological commitment itself dispensable. But this argument would fail if Field himself does not presuppose epistemological naturalism and that presupposition, so Katz seems to hold, would certainly beg the question if there is no supporting argument for it. Katz finds no such argument in Field's work. Field thinks that realists have "to postulate some aphysical connection, some mysterious grasping" and hence do not present any naturalistic account. Commitment to naturalism renders Field to go to the extent of holding positively the view that truths in mathematics are truths in a fiction. I think that Katz's refutation of this view is sound and that he succeeds in showing that there is essential difference between mathematics and fiction which goes to show that truth in mathematics cannot be taken to be truth in fiction. This difference lies in consistency being a necessary condition for truth in mathematics, while presence of inconsistency won't make fiction impossible. Those who indulge in fiction know that they are not doing mathematics and those who do mathematics know very well that it involves a serious logical exercise and that it's not a play of pure imagination. "Fiction is fiction and mathematics is fact".(p.14)

Against Classical Platonism as a kind of realism, he cites a move made by Godel in claiming that the relationship between ourselves and mathematical reality is not the kind of relationship that we have to physical reality in the case of sensations. Mathematical knowledge, he says explicitly, is not the causal effect of actions of certain things on our sense organs (pp.16-17), and suggests that Godel could not characterize the relationship because he had no epistemology to put in place of classical Platonism. Suggesting further that Godel is anything but a classical Platonist, he advises realists to take a cue from him for a different strategy in the task of integrating realism and rationalism. What one has to do is to extend the range of perceivable objects to include abstract objects. Contemporary Aristotelians seek to extend the range of perceivable to include the objects of mathematics but they locate the objects of mathematics in the natural world, and think that we have *a posteriori* knowl-

edge of sets just as we have of any material objects. He states and examines Maddy's position as a prominent example of attempt to avoid access to abstract objects in the Platonic realm on the basis of her idea that mathematical knowledge is knowledge by acquaintance. The strong Aristotelian import of her position is unable, it is claimed, to face some of the difficult philosophic questions that arise in the context of mathematical knowledge, truth and reality. Katz is also not happy with Naturalised realism either. Naturalised realism has a strategy to combine a realist ontology with an empirical epistemology resulting into a position which sounds too good to be true. Katz does not favour collapsing of formal sciences into psychology - an empirical investigation which leaves no scope for the study of abstract mathematical, logical and linguistic objects themselves. He examines the views of Alexander George and Colin McGinn and suggests that if the naturalistic knowledge of abstract reality requires non-naturalistic knowledge of such reality, the naturalised realism leaves the original problem of the knowledge of abstract objects exactly where it was. The moral which Katz gleans from the contemporary studies on philosophy of mathematics is that a realistic account of mathematical reality is possible only if realist avoids epistemologies based on i) acquaintance with abstract reality, and ii) acquaintance with a concrete reality. A realist must evolve an epistemology that treats knowledge of facts, laws and theories in the formal sciences as purely *a priori* knowledge - an epistemology that explains knowledge in the formal sciences on the basis of reason alone.

The epistemological challenge to realism, which is thus introduced in the first chapter, is taken up for detailed presentation and response in the second chapter entitled 'The Epistemic Challenge to Realism'. Paul Benacerraf's paper published in 1973, (*The Journal of Philosophy* pp. 661-80) is taken up for critical comments. It is argued in the first place that " although it seems clear that Benacerraf thinks that realism is unable to explain mathematical knowledge, his aim in this paper is not to refute realism" and that it's " to make both sides in the controversy.....face up to their problems" (p.25). In this chapter Katz tries to show that Benacerraf's



argument that mathematical knowledge is impossible if mathematical objects are beyond our causal reach is unsound. He claims that the argument is based on false assumption that information from causal interaction with natural objects is a necessary feature of justification in any form of knowledge. This assumption stems from the prevailing empiricist outlook which can be questioned. Empiricism can be replaced by a rationalist epistemology. Katz thinks seriously that realism has yet to receive a defense against the charge of epistemological malfeasance, and that he can provide such a long awaited defense. If realism rejects all forms of naturalism - ontological, epistemological and methodological -, its explanation of our knowledge of pure mathematics and other formal sciences should be based on the rationalist's notion that the truths of pure mathematics and other formal sciences are truths of pure reason. Benacerraf quite rightly claims that a realist's account of mathematical truth does not cohere well with an empirical account of knowledge but this would simply mean that in order to account for the real possibility of mathematical knowledge, we must have a non-empirical way of knowing mathematical truths. Katz makes distinction between mystery and mysticism and argues against all those antirealists who dismiss the possibility of any epistemology for abstract objects, thinking that it would involve some connection of a super-natural kind. There may be some philosophical mystery about how we come to acquire mathematical knowledge but it cannot be solved by adopting mysticism. The charge of mysticism is laid at the doors of realism because of the confusion between mystery and mysticism. Realism, when integrated with Rationalism, can resolve the mystery of mathematical knowledge with no mystical element in it. The rest of the chapter presents an outline of a Rationalist Epistemology taking cue from Godel. Mathematical intuition or the rational faculty on which mathematical knowledge rests cannot be associated with sensory data or something which is purely subjective. The possibility of mathematical knowledge indicates a different kind of relationship between the faculty we possess and the mathematical reality. "The task is to provide an account of this other kind of relationship that explains how we come to stand in that relationship to the realm of abstract objects."(p.34) Katz assumes, like Benacerraf, that

knowledge is justified true belief and analyses the belief condition, the truth condition and the justification condition so as to bring out their distinctiveness in the field of mathematics. The whole discussion reveals the confidence that in respect of the knowledge of formal sciences, there is no possibility of Gettier's problem ever raising its head. He makes two points: i) Reductio proofs in the formal sciences are tests of necessary truth, and ii) Reason is an appropriate instrument for determining how things must be in the realm of abstract objects of formal sciences. Taking the example of the proposition that two is the only even prime, he substantiates the points he makes. Further he draws attention to two aspects of reason that are responsible for the steps from knowledge of simple mathematical facts to knowledge of mathematical laws and theories. First, an order of knowledge for explaining the ascent from basic facts to knowledge of laws and theories, which characterizes both a priori and a posteriori knowledge- an ascent from basic knowledge to transcendent knowledge. While this feature of our knowledge in natural sciences can be easily shown to hold, thinkers have not paid enough attention to a similar distinction in respect of formal sciences. Second, In the formal sciences "it is common to refer to seeing that something is the case as "intuition" and to take such immediate apprehension as a source of basic mathematical knowledge."(p.43) A footnote to this last quote warns us not be misled by the use of the expression 'intuition'. There are genuine cases of basic mathematical knowledge that do not depend upon rational operations encompassed within a single grasp of structure. Katz takes care to ward off the cry of "mysticism" on the part of some radical antirealists but what is noteworthy is that he challenges Wittgenstein's criticisms which dismiss intuition as a source of knowledge referring to it as "an unnecessary shuffle".(PI:sec.213). It is Katz's view that caricature of intuition which Wittgenstein provides ignores the genuine role which intuition plays in mathematics, logic and linguistics. Even granting that intuition gives us sometimes wrong guidance, it is possible to integrate intuition into a systematic methodology that enables us to correct unclear and deceptive cases on the basis of a broad range of clear cases and principles derived from them. Katz makes it more than amply clear that the notion of intuition

that is relevant to the proposed rationalist epistemology is that of an immediate, non-inferential and purely rational apprehension of the structure of an abstract object. It is important to note that such an apprehension also reveals the limits of possibility with respect to the abstract objects having the structure. It is significant to note that Katz is an epistemic holist that has nothing to do with a semantic holism such as Quine's. It concerns the ways in which propositions in a particular system obtain their support from one another and from the basic knowledge on which the theory rests. He claims that "even though justification in formal sciences is *a priori*, propositions in those sciences are revisable in principle." (p.48) He rejects Quine's equation of *a priori* with unrevisability and also with analyticity. He also rejects the Quine-Putnam thesis that mathematics is legitimized in virtue of indispensability of numbers for natural science - the methodological naturalism. Acknowledgement of mathematical objects is not restricted to their role in natural science. He claims that we could establish their existence even if there were no empirical science. Mathematics and philosophy of mathematics are certainly older than natural science. The chapter ends with discussion of the question as to whether any questions have been begged - a question that has been answered in the negative. It also points to an all-important crucial distinction between natural and formal knowledge. Nature of the objects studied in the formal sciences is different from the nature of objects studied in natural sciences. "Investigation in the natural sciences seeks *to prune down the possible to the actual*, while investigation in the formal sciences seeks *to prune down the supposable to the necessary*."(p.59) while the former is *a posteriori*, the latter investigation is purely *a priori*. Given this vital difference between the two, it is clear that the expectation of the naturalists that for any ontological commitment we must have a perceptual contact with the object of knowledge cannot be fulfilled in respect of the abstract objects in formal sciences. It is the formal science that provides basis for positing existence of abstract objects. Our talk about, sets, numbers and other abstract objects in the field of mathematics and meta-mathematics becomes intelligible only if we understand it as a talk about abstract objects. Katz favours dualism of abstract and concrete entities and he claims that that

dualism is tenable. It's much better than the monism of ontological and epistemological naturalism and Quine's empirical way of getting his dualism off the hook.

In the Third chapter -The Epistemic Challenge to Antirealism -, Katz has an uphill task of countering the challenge posed by the naturalists and the empirically driven epistemologists to the special certainty of mathematical and other formal truths. Like Kant, Katz believes not only in the possibility of Mathematical knowledge but he also believes in the special certainty which it enjoys over all other kinds or species of knowledge. Mathematical knowledge is absolutely certain and is believed to be absolutely certain since Pythagorean times down to emergence of methodological naturalism of Quine and linguistic or philosophical naturalism of Wittgenstein. The onslaught of naturalism in the last few decades has strengthened the suspicions of some philosophers even in respect of mathematical knowledge to the point of the demise of their own discipline. Hume's skepticism did not dare to touch the zone of mathematics and Kant was happy to characterize this as 'fortunate error' on the part of that great inspirer of modern skepticism. Although Katz indicates his reasons for treating mathematical and logical truths to possess absolute certainty, he admits that it is still open for the naturalists to deny flatly the special certainty attributed to logical and mathematical truths. But they would not come forward to offer a satisfactory explanation of this feature of logical and mathematical truths on the grounds of full-fledged naturalism. Katz is not happy even with Quine's holism which treats total science as a single system saving empiricism from its uncompromising stand on mathematics as *a posteriori* knowledge. Katz sums up the present scenario saying that "since Quine, empiricists have stopped worrying about the certainty of mathematical and logical truths. "(p.66) He thinks that the two most influential forms of naturalism in contemporary philosophy are Quine's and Wittgenstein's. This is a position which he had taken in his earlier work of 1990, which I have already mentioned. Quine's naturalism is empiricist, scientific but dualist while Wittgenstein's is critically linguistic, ascientistic and monistic. What follows in the Section 3.3 and 3.4 is a very incisive criticism of Quine and Wittgenstein on the stances

which they have taken on giving an account of the special certainty of logical and mathematical truths. At the beginning of the 20th century, philosophy of mathematics was dominated by Frege's and Russell-Whitehead Logicism that attempted reduction of mathematical truths to logical truths but this, as it's wellknown, ultimately developed into a view that mathematical truth is convention. In 1936 itself, Quine had strongly criticised conventionalism, claiming that since logical truths are infinite in number, they can be grasped only as instance of more general principles and since we shall have to have some logic for this, conventionalism cannot offer explanation of logical truth. But this amounts to saying that true nature of mathematical truth eludes logicism and conventionalism implied by it. The analytic-synthetic distinction and the verificationist account of synonymy were criticised by Quine to build up an epistemology that could do a far better job of explaining the special certainty of logical and mathematical knowledge than any earlier radical empiricistic proposals. But although Quine is right in rejecting conventionalism, Katz thinks that "Quine's holistic conception of knowledge does not in the final analysis enable contemporary empiricists to provide a satisfactory account of the special certainty of logical and mathematical truth because the conception is inconsistent." (p. 72) He discusses the three constitutive principles (viz., non-contradiction, universal revisability and simplicity) of Quine's epistemology and shows how 'The Revisability Paradox' mars the prospects of uncompromising empiricism to meet the challenge posed before the antirealists by Katz. Katz emphatically argues that there can be no epistemology that says that everything including itself is revisable.

"Looked at from the right angle, universal revisability already flashes the signal Paradox! Paradox! Paradox! Unrestricted universality sanctions the dangerous move of self-application, which is a familiar feature of paradox.".....The paradox... undercuts the Quinean explanation of how truths of mathematics and truths of logic can be taken to be about natural objects in the Quinean sense of being a part of device for working a manageable structure into the flux of experience." (p. 74)

Criticism of Quine's holistic epistemology is followed by the criticism of Wittgenstein's naturalism. Wittgenstein in his *Philosophical Investigations* (1953) rejects, as we all know, philosophical theories and the metaphysical issues they concern. The traditional philosophy for him is not only bad but also non-sense resulting from the bewitchment of our intelligence by the snares of language. Thus for Wittgenstein, characterization of mathematical results as absolutely necessary are only a "somewhat hysterical way of putting the things" as held by him in his *Remarks on the Foundations* where he also maintains that the "must" that the mathematicians and philosophers typically use to express mathematical and logical truths is no more than "the expression of an attitude to the technique of calculation". As observed by Michael Dummett, Wittgenstein goes for a full-blooded conventionalism and for him the logical necessity of any statement is always the direct expression of a linguistic convention. Katz remarks that while Quine's position on mathematical and logical certainty is unacceptable logically Wittgenstein's is unacceptable linguistically. (p.76) The literal unintelligibility which Wittgenstein claims in respect of mathematical demonstration or for any logically compact proof is misleading. Neither mathematics nor Skepticism about mathematics is plain 'non-sense'. When we say that  $2+2=4$ , it is certainly a meaningful utterance and not a non-sense like saying 'colourless green ideas sleep furiously'. Wittgenstein's account indeed runs counter to the linguistic distinctions between meaningfulness and meaninglessness. Katz emphatically asserts that Wittgenstein was therefore wrong about certainty. It's the job of "pertinent specialists professionals" in linguistics to tell what to say about the meaningfulness or otherwise of sentences in natural languages, and not that of a philosopher.

Thus as against Quine and Wittgenstein, Katz claims that they do not have resources to face the challenge posed by mathematics, logic and linguistics - the formal sciences in general. Realism on the other hand faces the same challenge because it has the necessary resources. It explains the special certainty of formal truths in terms of their necessity and their necessity in terms of the abstractness of objects they are about.

Necessary formal truths are necessary because they describe unchangeable properties and relations of unchangeable objects. (p.78)

The 4th Chapter of the book under review is a reply to the semantic challenge to realism contained in Benacerraf's paper "What Numbers Could Not Be" published in 1965 which contains his famous argument for the indeterminacy of reference to numbers and symmetry claim about intended and deviant interpretations of arithmetic. Katz states Benacerraf's argument in brief as follows: "...there is no principled way of deciding which of the set-theoretic models of Peano arithmetic is the numbers, that there is no principled way of deciding what system of objects is the numbers, and hence we cannot make sense of the idea that numbers are determinate objects."(p.85) Benacerraf takes a structuralist view of arithmetic and considers it to be a science that elaborates the abstract structure that all progressions have in common merely in virtue of being progressions. It is not a science concerned with particular objects - the numbers. He ends his paper with the remark that "if the truth be known, there are no such things as numbers. " In a footnote, Katz makes it clear that although Benacerraf's paper does not reflect his present view on the subject, the paper itself is the classical statement of skepticism about the determinacy of reference to the numbers. The paper has long ago taken a philosophical life of its own, and thus has become a mile-stone in the development of skepticism concerning mathematical truth and knowledge. In order to show where and how exactly Benacerraf's argument is flawed, Katz has to use a very large canvas for analysing firstly two indeterminacy arguments in the philosophy of language - Quine's for indeterminacy thesis and Kripke's rule-following argument containing paradox which is of philosophical interest for everyone - that reveal their flaw, then for developing a conception of the structure of indeterminacy arguments in general and explaining how knowledge of the flaw in question can be used to develop a general strategy for resisting indeterminacy arguments. The canvas in the end shows that Benacerraf-style argument falls under the general conception of indeterminacy arguments. It also shows how to block such arguments. The justification for using wide canvas, as stated by Katz, is

that "linking indeterminacy arguments in the philosophy of mathematics and the philosophy of language provides a deeper understanding of the nature of such arguments and of the forms of skepticism based on them." (p. 87) The overall characterization of indeterminacy arguments is given as follows:

"Indeterminacy arguments are skeptical arguments. They claim that we lack the means to distinguish among the things we have to distinguish among in order to legitimize our belief that our talk about certain objects is talk about determinate objects. One characteristic of such skeptical arguments is that they are based on an allegedly unbreakable symmetry between the intended interpretation of such talk and certain deviant interpretations. The skeptic challenges us to break the symmetry." (p. 87)

One needs to read original arguments of Quine and Kripke very carefully and Katz's critical comments on them in original. Katz presents the debate between them on the one hand and himself on the other hand fairly well in all fairness. Katz's competence as a linguist can hardly be a matter of doubt. We should have no hesitation in accepting the view that the question of whether there can be a theory or theories of meaning is entirely a matter for linguist research to decide. Katz brings out the paradoxical element in both - the Quinean radical translation situation and the Kripkean rule-following situation - on the basis of his competence as a linguist. One should not forget that Quine almost always had Carnap in mind when thinking about semantics. Quine places senses out of the semantic picture and Kripke's puzzle also arises because senses have been painted out of the semantic picture. Once they are incorporated in the semantic picture in both respects, the finite extensionality and the finite intensionality, it is not difficult to overcome the puzzle that would embarrass any philosopher, who is unmindful of recent developments in linguistics.

This is followed by the discussion of the general form of Indeterminacy arguments. Here Katz is much more specific than the general charac-



terization of indeterminacy arguments we glean from the passage quoted above. The discussion is pin-pointed and serves as a prelude to the strategy for resisting indeterminacy. Katz spells out such a strategy and turns to Benacerraf's argument to show that it does not work. "Benacerraf's claim that The number words do not have single referents", says Katz, "follows only if number theory encompasses the full range of properties that can be used to exclude unintended models of arithmetic. Since it does not, his indeterminacy argument cannot take number theory as a complete explication of our knowledge of the numbers, and the alledged symmetry on which the argument rests can be rejected on the same grounds on which we rejected the alledged symmetries on which Quine's and Kripke's arguments rest. They underestimate our informal knowledge of the domain." (p. 110)

In the Section that follows, Katz has thrown his reflections on the 'metaphysics of Number-Theoretic Skepticism', though only sketchily. At the root of that skepticism lies the naturalistic view-point that philosophy has no subjectmatter of its own and that it is concerned merely with the semantic clarifications of ordinary and scientific language. Thus Benacerraf regards it as a mistake for philosophers to inquire into philosophical facts about numbers which fall outside the scope of mathematics proper. As Wittgenstein maintained that study of such facts is in vain as it's a pursuit of chimeras. Finally, Katz reacts to Hillary Putnam's paper 'Models and Reality' (published in *Journal of Symbolic Logic* (45), 1980, 464-82), which says that any philosopher or philosophically minded logician has to face insurmountable difficulty in viewing set theory as a description of a determinate independently existing reality because no formal system can ever capture our intuitive notion of a set. There is a Skolemite argument behind Putnam's move and it's Wittgenstein's rule-following argument that is at the root of it. Katz is not at all happy with Wittgenstein's posture, nor with Putnam's. Katz's conclusion is that determinate reference to the numbers and to the real world is possible and that it can be validated by our language, science and philosophy.

In the last but one chapter, Katz puts the realist in the docks to face

the Ontological challenge which, I suppose, is the most crucial stage in the whole argument of the book. The ontological challenge to realism is posed by certain alleged counterexamples to the traditional abstract/concrete distinction. Katz considers two kinds of counter-examples and attempts their explanation in a manner that saves for realist the distinction between abstract objects and concrete objects. It is true that realism cannot be formulated without the concept of an abstract object and if the alleged counter-examples are not taken care of, it would threaten to leave realism with no coherent formulation. Katz realizes that it is not quite easy to do this and that he has to present a long winding argument to meet the challenge before the realist.

Firstly, he sketches the conception of ontology underlying the reconstruction of the traditional abstract/concrete distinction. The intention is to understand the ontological issues properly and to extend the whole argument of the previous chapters to formulate rationalist-realist integration. He defines pure ontology as a foundational discipline of foundational disciplines, but clarifies that this conception is not foundationalism. Wittgenstein complained that since the results of mathematics are grounded in mathematical practice, they require no other grounding with the result that mathematics does not stand in need of philosophical foundations. Without raising any serious dispute about this view, Katz simply asserts that philosophy is not necessary to provide underpinnings for mathematical practice but only to provide understanding of it. (p.119)

"The aim of the philosophical foundations of a science is to shed light on ontological and epistemological aspects of the objects the science studies..... and not on the aspects of those objects with which the science itself is concerned." (p. 119)

"The foundations of mathematics, logic and other formal sciences are concerned *inter alia* with whether the reality studied in those sciences is abstract or concrete.....Foundational study of such foundational disciplines ... is concerned with understanding what it is to be abstract or concrete." (p.119)

Secondly, Katz reformulates the traditional distinction: abstract/concrete,

in the following way:

"An object is abstract just in case it lacks both spatial and temporal location and is homogeneous in this respect. An object is concrete just in case it has spatial or temporal location and is homogeneous in this respect".(p.120)

He defends atemporality, incorporeality, nonsensibility of abstract objects but rejects their transcendence in Platonic sense.

He further invokes the category of composite objects and argues that the supposed counter-examples belong to the category of composite objects and thus no longer pose challenge to realism. There is a fairly long discussion on the category of compound objects, a convincing plea for its recognition. The traditional distinction stands validated because we just can't think of formal sciences without committing ourselves ontologically to the existence of abstract objects. Use of Occam's razor may help us economize the plethora of such objects but the razor cannot clear all the abstract objects. In formal sciences, the question of using Occam's razor hardly ever crops up. Katz's contention is that since Occam's razor applies only in regard to evidence, it is as inapplicable to ontology as it is to mathematics, implying thereby that the category of composite objects that he espouses cannot be said to sin against that razor.

In the last chapter, Katz spells out his metaphilosophical reflections which broadly cover the entire scenario that has emerged owing to Frege's linguistic turn especially in the philosophical terrains of the anglo-saxon world. All these reflections are packed with deep and genuine awareness of the authenticity of philosophical problems. They have the strong potential to turn the entire tide against the positivistic, physicalistic, scientific and philosophistic forms of naturalism,(not to mention 'bald naturalism), and to restore the ontology of abstract objects in the area of Philosophy of mathematics. Afterall, Frege's intention in boosting Logicism with the famous Thesis of Extensionality of Language was to advocate Realistic Rationalism. He did want to integrate realism and rationalism. The course of philosophical argument during the 20th century has shown that that

integration was irrational. Katz traces the trouble in Frege's theory of meaning which says that the sense of an expression is to be explained in terms of its reference. Katz formulates an alternative to Fregean theory of meaning on the basis of his research in the field of linguistics. Intensionality of language cannot be dissolved into the extensionality of language. At no stage of his argument, I found Katz to be dogmatic or confused and unintelligible. I was happy to see that the road to metaphysics was not after all closed once for all. The gates are open and Katz has shown us the way. The main problem of Immanuel Kant's *'Transcendental Aesthetics'* viz., How is mathematics possible? has been answered afresh and one can say that this is a satisfactory answer until a more satisfactory one comes forward.

The entire argument of the book has been supplemented by illuminating footnotes from page to page which makes it extremely easier for readers to comprehend relevance of certain points and remarks made by the author in the course of his presentation. The detailed bibliography at the end will be helpful.

Jody Azzouni of the Tufts University has opined "that it is a brave book and readers will find in it (among other things) a full - scale assault on naturalism, a complex metaphysics of abstract, concrete and composite objects and a rationalist epistemology based on rich notions of Reason and Intuition", and Paolo Mancosu has said that "this book is certainly going to count as one of the most important contributions to the philosophy of mathematics of the last decades."

*Realistic Rationalism*, I shall not venture to say, is the last word on the nature of mathematics and mathematical knowledge. I shall say however, that it has brought to life the philosophical issues concerning them, which appeared to be written off to skepticism, naturalised or otherwise. In the words of Daniel Isaacson, "whether or not Katz's powerfully developed position is ultimately accepted, the terms of the debate have been very significantly advanced."