

**MODERN ANALYSIS OF SYLLOGISTIC LOGIC : A
CRITICAL REFLECTION**

The purpose of this paper is to examine modern analysis of syllogistic logic. It is concerned with one of the fundamental doctrines of syllogistic logic, namely, the doctrine of square of opposition. I have tried to show that modern analysis fails in refuting the Aristotelian doctrine. Instead of refuting the doctrine it rather supports the doctrine.

Aristotle, the founder of syllogistic logic, based his doctrine of square of opposition on existential import. He presumed that whenever categorical general propositions are used they do assert the existence of individuals. Aristotle's assumption of existential import ultimately led him to restrict his analysis only to those categorical general propositions whose terms all refer to non-empty classes; furthermore also to classes that have some members, but exclude all other propositions whose terms refer to empty classes, classes that have no members. Here one may ask this question : Why did Aristotle do so ? No doubt, there might have been some good reasons behind his assumption. But one of the reasons which comes to my mind at the moment is this : He must have thought that truth conditions for general propositions would never arise unless their terms refer to non-empty classes. That is to say, he must have thought that genuine predication presupposes the existence of something in the domain of discourse, otherwise the meaning of truth for quantified formulas becomes impossible. Thus it was his notion of truth

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which led him to base his doctrine of square of opposition on existential import. And, if this was the case, I think, he was absolutely right. Nonetheless, whatsoever other reasons there might have been, the fact remains the same, that is, his analysis was restricted only to those propositions whose terms all refer to non-empty classes. And other sorts of propositions for him were not genuine propositions from the semantic point of view. He divided all categorical general propositions into four categories, that is, A, E, I and O. A and E are universal propositions and I and O are particular propositions. Particular propositions are also called existential propositions. These propositions, according to him, stand to one another in certain relationships which Aristotle expresses as follows :

Thesis—1

A and E are contrary propositions. They might both be false but they cannot both be true.

Thesis—2

I and O are sub-contrary propositions. They can both be true but they cannot both be false.

Thesis—3

A and I, E and O are sub-altern propositions. The truth of A implies the truth of I and the truth E implies the truth of O but not *vice versa*.

Thesis—4

A and O, E and I are contradictory propositions. They can both neither be true nor false. If one of them is true, the other must be false.

For Aristotle, all these theses are valid. They express different types of relationships between two different categorical propositions,

Now, so far modern logicians, like Bertrand Russell and others, are concerned, they hold a different view on the matter. According to them, all relationships except contradiction are invalid. This they say because on their analysis A and E propositions can both be true and I and O propositions can both be false; and as a result of it neither the truth of A implies the truth of I nor the truth of E implies the truth of O. To substantiate their view modern logicians analyse the inner logical structure of categorical general propositions into symbolic language in the following way :

Propositions	Logical Forms or Structures
A	$(x) (\phi x \supset \psi x)$
E	$(x) (\phi x \supset \sim \psi x)$
I	$(\exists x) (\phi x \cdot \psi x)$
O	$(\exists x) (\phi x \cdot \sim \psi x)$

The logical structure of A and E clearly reveals two things : (a) A and E propositions are not categorical propositions. They are hypothetical propositions in their logical nature. (b) A and E propositions are quantifications of a complex propositional functions. So is the case with I and O propositions. As a result, whenever general propositions are used they do not assert anything about any individual. They assert the truth values of propositional functions. In fact, as per the modern analysis, individuals do not constitute as parts of the general propositions. General propositions are merely assertions about classes of functions, not of individuals. In this way, according to the

modern analysis, Aristotle's claim that general propositions do have existential import is unfounded. A and E both propositions are true in cases where $\emptyset x$ has no true substitution instances irrespective of what values are assigned to ψx , as they are universal quantifications of a complex propositional functions of the form $\emptyset x \supset \psi x$ and $\emptyset x \supset \sim \psi x$ respectively. No doubt, such cases on their view would arise only when expression 'x' in $\emptyset x$ refers to empty class, a class that has no member in it. It will not hold good in those cases where 'x' in $\emptyset x$ refers to non-empty class. Since A and E can both be true, therefore, according to the modern logicians, they are not contrary propositions at all. The thesis (1) is, therefore, an invalid thesis. Likewise, I and O both propositions are false in cases where $\emptyset x$ has no true substitution instances as they are existential quantifications of a complex propositional functions of the form $\emptyset x . \psi x$ and $\emptyset x . \sim \psi x$ respectively whose substitution instances are conjunctions with one of the false conjuncts. Since I and O can both be false in cases where $\emptyset x$ has no true substitution instances, therefore, according to modern logicians, they are not sub-contrary propositions at all. The thesis (2) is, therefore, an invalid thesis. As the thesis (1) and (2) are invalid so the thesis (3) is also invalid. The truth of universal proposition does not imply the truth of existential proposition. In this way modern logicians prove and substantiate their view that all relationships except contradiction are invalid. This they do by rejecting Aristotle's assumption of existential import.

No doubt, modern analysis is advantageous in the sense that it extends the scope of predicate logic by allowing the subject and the predicate terms of a proposition to refer both to empty and non-empty classes. But, then when we reflect critically on their analysis of propositions from the semantic point of view, we find that their analysis is not well grounded and generates

lots of problems. The reason why I consider modern analysis of propositions to be unsatisfactory from the semantic point of view is that there is a justification for it. Aristotle's conception of relation between two different propositions is embeded in their notions of truth and falsity. This is what I understand from their respective definitions. Had it not been so, Aristotle would not have excluded propositions whose terms refer to empty classes from his logical system of thought. He would have included such kind of propositions in his logical system of thought. But, since, as a matter of fact, he has excluded propositions whose terms refer to empty classes from his logical analysis, this, I think, itself is an evidence to say that Aristotle's doctrine of square of opposition is grounded in the notion of truth and falsity of propositions. This, he thinks, is not possible unless it is admitted that the universe of discourse is non-empty (under which propositions are evaluated in terms of their truth and falsity). If this is so, the analysans and analysandum of modern analysis must depict the same logical property; otherwise, their analysis becomes futile and incorrect. This is the logical requirement of any analysis to be a correct analysis. But unfortunately a critical reflection of their analysis shows that it has failed to fulfil the logical requirement. Instead of refuting Aristotle's doctrine it rather supports his doctrine. How modern analysis has failed in refuting Aristotle's doctrine can be shown in the following way :

Consider for example, the symbolic expressions of (x) $(\phi x \supset \psi x)$ and (x) $(\phi x \supset \sim \psi x)$. These expressions are the logical forms of A and E propositions respectively. They show that A and E propositions are quantifications of complex propositional functions of the form $\phi x \supset \psi x$ and $\phi x \supset \sim \psi x$. Expressions $\phi x \supset \psi x$ and $\phi x \supset \sim \psi x$ are expressions of propositional functions because their components ϕx and ψx are propositional functions as they contain individual variable 'x' in

them. Since individual variables are undetermined expressions, $\phi x \supset \psi x$ and $\phi x \supset \sim \psi x$ expressions are also undermined expressions. As a result of it, they fall short of the categories of truth values. They cannot be characterized as true or false. Here, it is worth noting that when it is said that propositional functions fall short of the categories of truth values it does not mean that their values also fall short of the categories of truth values. The values of propositional functions can be either true or false since they take singular propositions as their values whose elements are all determined. But this is not equivalent to saying that propositional functions are either true or false. What is true of a proposition is not true of propositional function and *vice versa*. Since propositional functions, $\phi x \supset \psi x$ and $\phi x \supset \sim \psi x$ fall short of the categories of truth values and they can neither be true nor false. But A and E propositions also fall short of the categories of truth values. They cannot be characterised as true or false (speaking strictly), as they are quantifications of complex propositional functions. And if A and E propositions cannot be either true or false, then it makes no sense to say that A and E are true in cases where their subject terms refer to empty classes, as modern logicians claim. Consequently, Aristotle's thesis (1) stands unrefuted. The same is true of all other theses. Mere quantification of the variable of a propositional function does not solve the issue. For quantifying the variable of a propositional function is not rendering of that propositional function true or false. Quantification is not the criterion of truth values. In other words, to quantify an individual variable of a propositional function is not to assign truth values to that propositional function. Whenever we quantify any individual variable of any propositional function, what in fact is done is this : We indicate the number of values of individual variable (the values of individual variables are individuals); beyond this nothing is done. In other words, when we quantify any individual

variable of any propositional function we make the variable bound, that is, we bring the variable under the scope of quantifier. And to do so is not to change that variable into constant. The essential nature of variable remains in the process unchanged. The only difference between them is that the values of a quantified variable are determined whereas the values of an unquantified variable are undetermined. And to do so is neither to assign the truth values to propositional function whose variable are quantified nor to change its variables into constants. Quantification of the values of a variable of a propositional function is one thing and to assign truth values to the propositional function is another. The criterion of the one is not the criterion of the other. Quantification is a mere fixation of the values of a variable of a propositional function, not the truth value assignment to propositional function. If someone thinks so, he is under a wrong impression. Thus, what follows from the above discussions is this : That general propositions, upon the modern analysis, cannot be either true or false. This is true in the light of their logical forms. If general propositions cannot be either true or false, then it follows therefrom that modern logicians have failed in refuting the Aristotelian doctrine of square of opposition, as its theses are embedded in the truth values of A, E, I and O propositions.

It might be argued against my view that when modern logicians say quantified formulas or propositions are true or false they do not mean to say that they are true or false in themselves; rather they are true or false of individuals. If a propositional function of a quantified formula is true of an individual, the individual, on their view, satisfies that function and consequently it makes formula true. Likewise, if a propositional function of quantified formula is not true of an individual, the individual does not satisfy that function and consequently it renders formula

false. In this way modern logicians, instead of assigning the truth values to quantified formulas themselves, judge their validity by interpreting them in terms of their substitution instances in which individual constants are put in for individual variables. For example, modern logicians interpret universally and existentially quantified propositions respectively as conjunction and disjunction of singular propositions. The rationale behind their interpretation of quantifiers as conjunction and disjunction is this : They think there is a similarity of logical behaviour between universal and existential quantifiers on the one hand and the logical constants of dot ‘.’ and vel ‘V’ on the other. As a result of it, they lay down the following criterion of truth for quantifiers : A universal formula, say for example, $(x) (\phi x \supset \psi x)$ is true iff its substitution instances, for example, $(\phi a \supset \psi a)$, $(\phi b \supset \psi b)$, $(\phi c \supset \psi c)$, ... $(\phi n \supset \psi n)$ are all true; otherwise, the formula is false. Similarly, an existential formula, say $(\exists x) (\phi x \cdot \psi x)$, is true iff its at least one substitution instance, for example, $(\phi a \cdot \psi a) \vee (\phi b \cdot \psi b) \vee \dots (\phi n \cdot \psi n)$, is true; otherwise, the formula is false. Thus, what follows from the above discussion is this : A, E, I and O propositions are, according to the modern logicians, not true and false in themselves, but they are true and false of individuals; that is, their interpretations are true or false. Therefore, it can very well be maintained that modern analysis refutes Aristotle’s doctrine of square of opposition. My humble submission here would be this : Even if we hold that quantified formulas or propositions are not true or false in themselves but that they are true or false of individuals, it does not in any way contradict Aristotle’s doctrine. It rather supports his doctrine. Because quantifier’s interpretation itself presupposes that the universe of discourse is non-empty. There exists at least some individuals of which quantified formula or proposition is true or false. Besides this, there is another presupposition : Universal quantifier and conjunction, existential

quantifier and disjunction are logically equivalent; otherwise, testing the validity of quantifiers on the basis of the truth values of conjunction and disjunction would logically become impossible. But the presupposition that universal quantifier and conjunction, existential quantifier and disjunction are logically equivalent implies not only that quantifiers and logical constants of dot '·' and vel 'v' do have logical similarity (if not identity) but also that a quantifier must be either true or false. Because conjunction and disjunction in which quantifiers are interpreted are truth functional in character and presuppose two-valued logic. This means whatever propositions we use in logical operators of dot '·' and vel 'v' must have one of the truth values, true or false in complete sense. But to accept this view would amount to reject modern analysis which advocates the thesis that quantified formulas or propositions are not true or false in themselves as they involve propositional functions in them. Besides this, acceptance of presupposition also reduces class-notion to its members; whereas the fact is otherwise. A class is not a member of itself. What is true of a class is not true of its member and what is true of member of a class is not true of that class itself. These consequences are certainly unacceptable to the modern logicians. It cannot be said that quantifier's interpretation does not necessarily presuppose the discourse of the domain of non-empty. Had it been the case, modern logicians would not have advocated the view that quantified formulas are not true or false in themselves on the ground of propositional functions involved in them. They would have set forth the truth conditions for quantified formulas and propositions directly like those for truth functional formulas and propositions without interpreting them in terms of truth functional language, that is, conjunction and disjunction of singular propositions. But they do not do so. This itself is a good evidence to say that they have based their

quantifier's interpretation on the assumption that the domain of discourse (under which quantifiers are evaluated as true or false of individuals) is non-empty. But the acceptance of this view would amount to the acceptance of the validity of Aristotle's doctrine of square of opposition which modern logicians refute. Nonetheless, the whole discussion reflects that modern logicians are not consistent in their logical analysis of A, E, I and O propositions of Aristotelian logic.

What I have said can also be seen to stand valid from another angle : According to the modern logicians universal propositions are conjunction of singular propositions and existential propositions are disjunction of singular propositions. Singular propositions are the simplest kind of propositions out of which their conjunctions and disjunctions are constructed. Neither do they contain any truth functional operators nor any quantifiers. But all this does not mean that singular propositions do not have truth values. Singular propositions do have truth values for the simple reason that their subject terms, being logically proper names, always stand for some individuals of which the predicates of propositions either ascribe or deny certain properties. In other words, a singular proposition always asserts that a certain object specified by its logical subject called proper name satisfies or does not satisfy certain property specified by the predicate of that proposition. If the object referred to does in fact satisfy the description, the singular proposition is true or else it is false. This way of interpreting any singular proposition clearly shows that it is the predicate of proposition that is true or false of what the subject of proposition stands for. In other words, it is the function of predicate that yields true or false proposition, not the other way round. We can get the negation of subject-predicate proposition by simply replacing the predicate term by its own negation, whereas

we cannot do this with the subject. But to accept this line of interpretation of a singular proposition would amount to admit that Aristotle's doctrine of square of opposition is valid. This is so because singular proposition's interpretation not only excludes propositions whose terms refer to empty classes from the categorization of truth and falsity but also establishes positively that truth values of their conjunctions and disjunctions in which quantifiers are interpreted, presuppose the universe of discourse being non-empty. There exists at least an object in the universe of discourse referred to by the logical subject of propositions which satisfies or does not satisfy a certain property, specified by the logical predicate of that proposition. And this is exactly what I think Aristotle also had in mind when he based his doctrine on existential import. He thought that the meaning of truth for quantified proposition would never arise unless it is granted that the domain of discourse is non-empty. But to admit this view would amount to admit that general propositions whose subject terms refer to empty classes lack truth values. This, however, the modern logicians deny. Because empty classes contain no individuals of which something can be predicated positively or negatively. Meaningful predication of a certain property to something always presupposes, from the semantic point of view, the existence of that thing in the domain of discourse whose absence always produces truth valuelessness, unless we grant the existence of non-existent entities would mean to admit that the same fact does not make one proposition true and the other false. As a result of this, we would be compelled to discard the rule of negation which states that negation of a true statement is false and negation of a false statement is true. Therefore, given these two alternatives, I think, it is better to admit that absence of an object from the entire domain of discourse produces truth valuelessness instead of admitting the existence of non-existent entities. Here, it is worth noting that

there is a distinction between an object being absent from the entire domain of discourse and an object being absent from a particular part of the domain of discourse. The latter does not produce truth valuelessness, as it implies that the object in question does exist in some other part of the domain of discourse, whereas the former produces truth-valuelessness, as it implies that the object in question exists nowhere in the entire domain of discourse. That is, it is a non-existent entity. If a term is empty it fails to refer, talking from the logical point of view, to anything and if it refers to something genuinely, it ceases to be an empty term. This is primarily because of the fact that emptiness is incompatible with referent. Thus, what makes a proposition true or false is not the presupposition of the existence of an object specified by the subject term of proposition but the actual fact. The criterion of truth is independent of speaker's beliefs and presuppositions. Its source lies in reality.

An objection may be proffered against my view that when modern logicians refute Aristotle's doctrine of square of opposition they refute not on this ground that it is the predicate that always yields true or false proposition but rather on the ground that predicates do not always yield true or false propositions. Because there are propositions, according to them, whose truth values do not require any existential import. Such type of propositions are those whose subject terms refer to empty classes. Consider, for example, quantified formulas $(x) (\phi x \supset \psi x)$ and $(x) (\phi x \supset \sim \psi x)$. These formulas are the formulas of A and E propositions respectively. In this regard, so far Aristotle's view is concerned, these formulas are contrary formulas as their exemplifications might both be false, but they cannot both be true. Aristotle arrived at this conclusion because of his assumption of existential import. But so far as the modern view is concerned, these formulas are not contrary as their exemplifica-

tions are true in cases where the subject expression 'x' in ϕx happens to be an expression of empty class. Therefore, according to the modern logicians, it is incorrect to say that predicates always yield true or false propositions. They yield, on their view, true or false propositions only in the those cases where they are attached to non-empty subject expressions of propositions. They fail in cases where subject expressions of propositions stand for empty classes; for example, "The unicorns are animals". In the latter cases it is the subject of proposition that makes the proposition false without any existential requirement. Proposition becomes false not because of false predication but because of its subject being empty. Against this my humble submission would be that even if the latter criterion of truth is adopted, it does not refute Aristotle's doctrine of square of opposition on the following grounds :

- i) If emptiness of a class referred to by the subject term of a proposition makes proposition false, irrespective of what is predicated by the predicate term of that proposition, then I think its opposite thesis must hold good equally, that is, non-emptiness of a class referred to by the subject term of a proposition must render that proposition true, irrespective of what is predicated by the predicate term of that proposition. This follows from the dichotomy of truth and falsity. But this is not true as a matter of fact. For example, the subject term of the proposition "Mohan is the son of a barren woman" is a non-empty term and yet the proposition is not true. Modern logicians themselves would not hold the view that non-emptiness of a term renders proposition true. But they are compelled to admit this by their criterion of truth value. This shows that they are not consistent in their analysis and hence their criterion of truth value must be rejected.

- ii) Even if it is accepted that emptiness of a class referred to by the subject term of a proposition makes that proposition false it does not invalidate Aristotle's doctrine of square of opposition because Aristotle's criterion of truth value was based on the predicate of a proposition, not the subject of that proposition. The subject-criterion of falsity (the criterion based on the subject of a proposition I call the subject criterion of truth value) is distinct and different from the predicate criterion of falsity (the criterion based on the predicate of a proposition I call the predicate criterion of truth value). Since both these criteria differ in their logical ground, and Aristotle's doctrine of square of opposition is based on the predicate-criterion of truth value; therefore, his doctrine cannot be invalidated on the ground of subject-criterion of truth value. Aristotle's doctrine would have become invalid had it been shown by the modern logicians that A and E proposition are true and I and O propositions are false in certain cases on the ground of predicate criterion of truth value, not on the ground of subject criterion of truth value. This modern logicians have not done. Therefore, Aristotle's doctrine still stands valid, even if it is accepted that emptiness of a class referred to by the subject of a proposition makes the proposition false. Both the criteria are not incompatible as they differ in their logical grounds.
- iii) Although modern logicians adopt different criterion of truth value than that of Aristotle in refuting his doctrine of square of opposition, yet so far as the symbolization of of both types of propositions is concerned, that is, propositions whose subject terms refer to non-empty classes and propositions whose subject terms refer to empty classes, they do not maintain the distinction. They symbolize both

the subject-negation and the predicate-negation in the same fashion. As a result of it, instead of offering the solution to the problem, they create more confusions. Their analysis of proposition is not neat and clean.

- iv) Above all, truth and falsity are a kind of predicates that are predicated to propositions. This is what generally is accepted. Now, if emptiness of a class referred to by the subject term of a proposition makes, on the modern view, that proposition false then it amounts to saying that it is the subject not the predicate that is false of a certain object specified by the subject of a proposition, irrespective of whether any predicate is predicated to that object or not. But to accept this view would not only compel them to treat the subject and the predicate alike in their logical status but also to equate referring function with describing function. This is because the subject term of a proposition which refers to nothing attributes the property of falsity with which modern logicians will never agree. But nonetheless it follows from their subject-criterion of falsity.

Thus, in short, from the above discussions we find that modern logicians have not succeeded in refuting Aristotle's doctrine of square of opposition. Their analysis itself defeats their objective.

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