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LOGICAL PRIMITIVES

P. Geach has often claimed that such applicatives as 'any', 'every', 'some', 'a(n)', 'no', 'most', and 'only' belong to the same syntactical category, viz. quantifiers.[1] One of his pet theses has been that such signs, when attached to general terms do not form referential expressions. For if 'E' is a referential expression then it is always appropriate to ask its user to specify to *which* individual he refers. Since this is rarely possible for expressions like 'some A', and senseless for expressions like 'no A' (consider: 'No men live on the moon'. 'Which men don't live on the moon?'), the argument is that *no* such expressions (applicative plus general term) are referential.

F. Sommers has effectively challenged these Geachian claims.[2] In particular, Sommers has argued that logically quantified expressions *are* referential, and since not all applicatives listed above form referential expressions when attached to terms, not all of those applicatives are logical quantifiers. Indeed, to put Sommers' thesis concerning logical quantifiers most bluntly, there is but one logically primitive quantifier, which in English is usually rendered by such applicatives as 'some' and 'a(n)'. The rest are either special duty versions of the logically primitive quantifier, or are reducible or definable in terms of it plus other logical signs. Thus: 'any' is the primitive quantifier in distributing contexts (e.g. negations, antecedents), 'no' is defined in terms of the primitive quantifier and sentence denial, 'every' is defined in terms of 'no' and term negation, 'most' is given a still more complex definition.[3]

Of course Sommers' quantifier thesis depends upon his ability to say just what the logically primitive formative signs are. The logically primitive formatives of his logic are the quantifier and two kinds of negation and sentential conjunction. One kind of negation operates on terms to form their logical contraries. Such an operation reflects the contrast in English between members of such pairs as 'red'/'nonred', 'married'/'unmarried', 'male'/'nonmale'. The second kind of negation operates on entire sentences to form their contradictories. Since in Sommers' symbolic calculus both negations are indicated by the same sign, '–', ambiguity is resolved by position. The quantifier is '+'. The sign of conjunction, 'both...and...', is rendered by the pair '+...+...'. The nonformatives, or variables, of Sommers' system are of two kinds: term variables (uppercase letters) and sentential variables (lowercase letters). Sentences containing only term variables as nonformatives are *categoricals,* others are *compounds.* Unnegated terms and unnegated sentences are either marked with a '+' or left unmarked (as in algebra) where no ambiguity would arise.

The simplest categoricals are sentences sharing the forms of 'Some S is P', 'Some S is nonP' or their negations, 'Not: some S is P' and 'Not: some S is nonP'. These last two can be reformulated as 'No S is P' and 'No S is nonP' revealing that 'No' is a portmanteau word defined by 'Not: some'. 'No', then, is no quantifier (contra Geach), but instead is a polyfunctor combining the roles of sentence negator and quantifier. This is easily seen in Sommers' notations for the four sentence forms above.

Sentence	*Full notation*	*Simplified notation*
Some S is P	+(+(+S)(+P))	+S+P
Some S is nonP	+(+(+S)(–P))	+S–P
Not: some S is P	–(+(+S)(+P))	–(+S+P)
Not: some S is nonP	–(+(+S)(–P))	–(+S–P)

In the full notation the first formative sign is that of sentential affirmation or denial (negation), the second is the quantifier, and the third and fourth indicate in each case whether the term is negated or unnegated. Obviously 'no' is not a quantifier. It's notational representation is complex, viz. '–(+......)', indicating its dual function.

'Every' is a quantifier. But it is not a logically primitive one. Again, in Sommers' system there is but one logically primitive sign of quantity — the "particular" quantifier. So 'every', the "universal" quantifier, must be a nonprimitive formative, definable in terms of primitive formatives. Consider a sentence of the general form 'Every S is P'. If we equate this with 'No S is nonP' it can be symbolized as: '–(+(S) (–P))', given our definition of 'no'. Let us simplify this as: '–(+S–P)'. If we now algebraically distribute the external sentential negation we get: '+(–S+P)', or simply: '–S+P'. Thus 'every' is marked in Sommers' system by '–'. The universal quantifier is not primitive, but it is defined in terms of sentence negation, particular quantity, and a term negation. Its symbolic representation, '–' (in quantifier position) is always replaceable by '–(+....–...)'.

One applicative which exercised Geach but about which Sommers has said little is 'only'. Nevertheless, 'only' presents no difficulties for Sommers' theory of logical primitives.[4] The word 'only' occurs in logically important positions in both categoricals (e.g. 'Only S is P') and compounds ('p only if q'). I begin with its categorical version. Sentences of the general form 'Only S is P' (e.g. 'Only men serve as generals', 'Only dogs bark', 'Only gods are immortal') are equivalent to sentences of the general form 'No nonS is P'. And, as we have seen, 'no' is analyzable as 'Not: some...'. So 'only S is P' is equivalent to 'Not: some nonS is P'. 'Only', then, is a polyfunctor combining at once the roles of sentence negator, quantifier, and term negator. We could symbolize 'Only S is P' as '–(+(–S) (+P)), revealing that 'only' is a triple function symbolized as '–(+(–.. ....))', (i.e. 'not: some non....'). Compare: 'Not an unmarried woman was at the party'/'only married women were at the party'.

We have just seen that in categoricals 'only' plays at once the roles of three primitive formatives — two negations flanking a quantifier. Its tri-functional status there is paralleled by its status in compounds. In the past some logicians have worried over the relationship between categoricals and compounds (and, perforce, over the relationship between a logic of terms and a logic of sentences). Early syllogists, following Aristotle's lead, were generally content to either ignore compounds or simply say very little about them. They tended to hint that compounds could be treated as categoricals. The Stoics ofcourse held that compounds deserved to be treated on their own and were not reducible in any way to categoricals. Leibniz argued that compounds must be reduced to categoricals. And Boole (and even Frege)[5] made similar suggestions. Contemporary mathematical logicians, on the other hand, tend to take the logic of compounds as more primitive than the logic of categoricals. Peirce[6] held that neither was reducible to the other, and after initially holding a Leibnizian position Sommers has come to this Peircean view as well. According to this view categoricals and compounds are mutually irreducible but structurally isomorphic, sharing common underlying logical forms. Sommers reflects this isomorphism by using a notational system which permits either categorical or compound interpretations. The logically primitive formatives for categoricals are the (particular) quantifier (symbolized by '+'), sentence negation ('–'), and term negation ('–'). The simplest categoricals have the general forms:

Some S is P
Some S is nonP
Not: some S is P
Not: some S is nonP

The logically primitive formatives for compounds are sentential negation ('–'), and the binary connective 'both... and...', symbolized by a pair of signs ('+...+...'). The simplest compounds have the general forms:

Sentence	*Full notation*	*Simplified notation*
Both p and q	+(+(+p) + (+q))	+p+q
Both p and not q	+(+(+p) + (−q))	+p−q
Not: both p and q	−(+(+p) + (+q))	−(+p+q)
Not: both p and not q	−(+(+p) + (−q))	−(+p−q)

Other compounds are either not simple (i.e. contain additional occurances of primitive formatives, e.g. 'Both not p and not q') or are definable in terms of more primitive compounds. Thus, for example, 'If p then q' is definable as 'Not: both p and not q'. By algebraically distributing the external negation of its formula ('-(+p-q)') we get '-p+q'. While 'both...and...' ('+...+...') is primitive, 'if... then...' ('-...+...') is not. Instead the conditional is defined in terms of the two primitives: sentential negation and conjunction. Likewise, 'p unless q' is definable as 'Not both not p and not q'.

Consider now a compound of the general form 'p only if q'. We could reformulate this as 'Only if q, p'. And this is equivalent to 'Not: both not q and p'. Notice the parallel here between:

Not: some nonS is P

− (+ (− S) (+ P))

and

Not: both not q and p

− (+ (− q) + p)

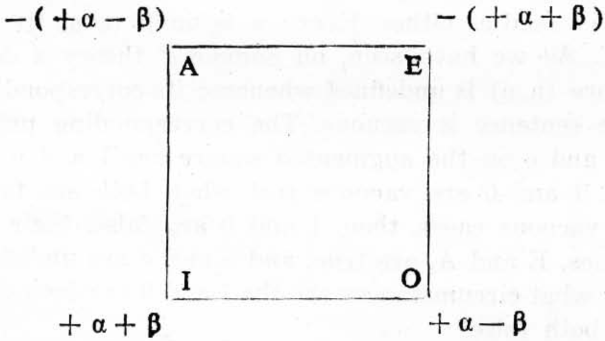
In each case the 'only' expression is expandible into a '−(+ (−...)' expression. By using neutral variables, interpretable either as terms or as sentences (e.g. '∝', 'ß', etc.) a formula such as '−(+(−∝)+ß)' could be read either as 'Only ∝ is ß' or as 'ß only if ∝'. The expression 'only if' is not a primitive sentential connective. Rather it is a triple functor combining the roles of sentence negation, sentence conjunction, and a second sentence negation.

Notice that the sentence 'Not: both not q and p' is equivalent, given the commutibility of conjunction, to 'Not: both p and not q', a simple compound. Let us distinguish between *primitive* formatives (e.g. 'some', 'not', 'non', 'both...and...'), *definable* formatives (e.g. 'every', 'if... then...'), and *reducible* formatives (e.g. 'no', 'only', 'unless'). Definable formatives are given their own symbolization (thus 'every' is symbolized by '–'), which can be defined in terms of primitive formatives. Reducible formatives are not given a special sybolization but are seen as polyfunctors always symbolized as combinations of two or more different primitive formatives. So while 'every' is symbolized by '–' (and is definable as '–(+...–...)'), 'no' is not given a special symbol but is always replaced by 'not: some' ('–(+...)').

Why is it important to distinguish a special, privileged class of primitive logical formatives as Sommers has done? One reason is theoretical economy. This is the kind of economy that a propositional calculus using the Sheffer stroke has relative to the standard notational system with a number of propositional connectives. But a more important reason is this. There are circumstances in which definable, nonprimitive formatives are in fact undefined. On those occasions sentences containing such formatives are uninterpretable and can be assigned no truth-value. This is never the case for elementary sentences containing only logically primitive formatives or for reducible sentences containing only primitive or reducible formatives.

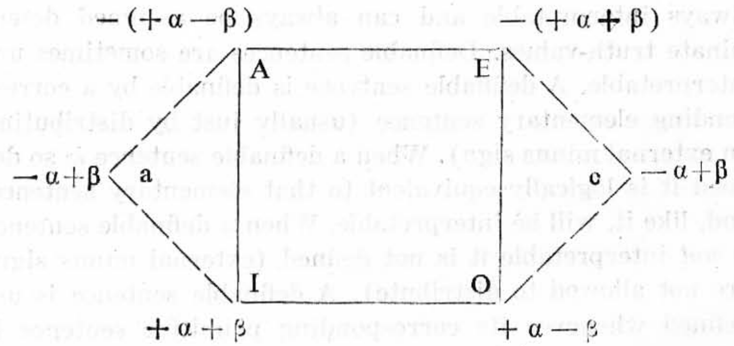
We are calling a sentence with only primitive formatives a *primitive sentence*, and a sentence with only primitive and reducible formatives a *reducible sentence*. We will call a sentence with at least one definable formative a *definable sentence*. Further, let us call all primitive and reducible sentences *elementary sentences*. Elementary sentences are always interpretable and can always be assigned determinate truth-values. Definable sentences are sometimes uninterpretable. A definable sentence is definable by a corresponding elementary sentence (usually just by distributing an external minus sign). When a definable sentence *is* so defined it is logically equivalent to that elementary sentence, and, like it, will be interpretable. When a definable sentence is *not* interpretable it is not defined (external minus signs are not allowed to distribute). A definable sentence is undefined whenever its corresponding primitive sentence is *vacuous*.

To see what is meant by the "corresponding primitive sentence" of a definable sentence let us construct a square of opposition.



Here the variables can be read as terms, to yield categoricals (I: 'Some α is β'), or as sentences, to yield compounds (I: 'Both α and β'). Notice that I and O are primitive, and A and E are reducible. All are elementary. We can refer to this as an *elementary square.* We can read A on this square in a variety of ways: 'Not some α is nonβ', 'No α is nonβ', 'Only β is α', 'Not: both α and not β'. Corresponding readings (e.g. 'No α is β') are available for E.

Definable sentences are not found on the elementary square. To display them we need an extended, *augmented square.*[7]



Here a and e are definable sentences. When defined a is equivalent to A and e is equivalent to E (this is simply obversion, which in Sommers' system amounts to external minus distribution). A sentence of the form a could be read either as 'Every ∝ is ß' or as 'If ∝ then ß'. And e could be read as either 'Every ∝ is nonß' or as 'If ∝ then not ß'. As we have seen, on Sommers' theory a definable sentence (a, e) is undefined whenever its corresponding primitive sentence is vacuous. The corresponding primitives for a and e on the augmented square are I and 0 respectively. I and 0 are vacuous just when *both* are false.[8] In these vacuous cases, then, I and 0 are false, their contradictories, E and A, are true, and a and e are undefined. So under what circumstances are the I and 0 versions of a sentence both false?

Sommers has provided arguments,[9] which need not be repeated here, to show that among the vacuous cases are those in which the subject-term of a categorical is empty (e.g. 'Some female U.S. president is married/unmarried'), those in which the categorical is a category mistake[10] (e.g. 'Some numbers are married/unmarried'), those in which the categorical's subject is undetermined with respect to the predicate (e.g. 'Some man will/won't live on Venus in the Twenty-second Century'), and primitive compounds whose first conjunct is false (e.g. 'Both wood is denser than gold and silver is/is not denser than wood'). In each of these cases the I and 0 sentences are arguably false (so their E and A contradictories are true). But the corresponding a and e sentences are undefined. Thus, for example, all of the following would be uninterpreted sentences.

1. Every female U.S. president is married.
2. Every female U.S. president is unmarried.
3. Every number is married.
4. Every number is unmarried.
5. Every man will live on Venus in the Twenty-second Century.
6. Every man will fail to live on Venus in the Twenty-second Century.
7. If wood is denser than gold then silver is denser than wood.
8. If wood is denser than gold then silver is not denser than wood.

Most modern mathematical logicians are forced by their standard system to say that the eight sentences above (with possible doubts about 5 and 6) are true. Sommers' has developed a system of logic which in effect avoids such paradoxical consequences by distinguishing between definable and elementary sentences, a distinction which rests ultimately upon the distinction between logically defined and logically primitive formatives.

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NOTES

1. See *Reference and Generality* (Ithaca, 1962), esp. chapter 1; *Logic Matters* (Oxford, 1972), esp. 1.1, 1.5, and 3.8; "Distribution and *Suppositio*", *Mind*, 85 (1976); and *Reason and Argument* (Berkeley, 1976).
2. He has done this in several places, most recently and extensively in *The Logic of Natural Language* (Oxford, 1982).
3. See Appendix D of Sommers' 1982.
4. In 1983 I read an unpublished paper, "Only", by V. Balowitz. While I disagree with much that he says there, my analysis of 'only' is essentially the same as his. He also correctly points out that the standard view that 'only' is a dual functor which at once quantifies and reverses term positions is mistaken.
5. See Sommers' "On Concepts of Truth in Natural Languages", *Review of Metaphysics*, 23 (1969), note 3 (pp. 265-6).
6. See R. Dipert, "Peirce's Propositional Logic", *Review of Metaphysics*, 35 (1981).
7. See the following series by G. Englebretsen: "Trivalence and Absurdity", *Philosophical Papers*, 4 (1975); "The Square of Opposition", *Notre Dame Journal of Formal Logic*, 17 (1976); "Opposition", *Notre Dame Journal of Formal Logic*, 25 (1984); and "Quadratum Auctum", *Logique et Analyse*, forthcoming.
8. In addition to the items listed in the preceding note see: F. Sommers, "Predicability", *Philosophy in America*, ed. M. Black (Ithaca, 1965); G. Englebretsen, "Vacuousity", *Mind*, 81 (1972); and C. Sayward and S. Voss, "Absurdity and Spanning", *Philosophia*, 2 (1972).
9. See especially chapter 14 of Sommers' 1982.
10. For Sommers on category mistakes see: "Types and Ontology", *Philosophical Review*, 72 (1973); "Predicability", *loc. cit.*; and chapter 13 of his 1982. For a summary see G. Englebretsen, "A Reintroduction to Sommers' Tree Theory", forthcoming.