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THE IDEA OF MOTION*

At least some of the philosophical controversies have arisen as a result of people having made statements in the ontological idiom where only a linguistic one would have been justified, or, what is here the same, having made categorical assertions where only a conditional statement could legitimately have been made. Many a philosophical argument which appears to us absurd, silly, or invalid, can be made analytically true by simply turning categorical assertions into consequents of hypothetical propositions, or alternants in disjunctive/alternative propositions. No philosopher, not even a philosopher of Russell's calibre and persuasion the greater part of whose philosophical writings had been a crusade against the consequences of the above mentioned practice, has succeeded in withstanding the gravitational pull excercised by the ontological idiom.

One such example is afforded by the controversy between those who accepted and those who rejected the at-at notion of motion (i.e. the view that a body occupies or coincides with a space equal to its volume even while it is in motion, motion consisting in the mere fact that the body is in different places at different times). Russell asserted,

> 'People used to think when a thing changes, it must be in a state of change and that when a thing moves, it is in a state of motion. This is now known to be a mistake. Motion consists merely in the fact that bodies are sometimes in one place and sometimes in another and that they are at intermediate times.'[1]

to which Bergson may be said to have retorted,

> '...with immobility set beside immobility, even endlessly, we could never make movement.'[2]
>
> 'Take the flying arrow..... Motionless in each point of its course, it is motionless during all the time that it is moving. Yes, if we suppose that the arrow can ever be in a point of its course. Yes again, if the arrow, which

is moving, ever coincides with a position which is motionless. But the arrow never is in any point of its course.'[3]

Regarding Russell's statement, one is tempted to ask, what new fact of nature has been discovered by scientists to oblige people to think differently than they have been wont to, and if no such fact is shown to have been discovered, then one would like to know what had prompted Russell to make such an assertion? — and would not his statement ('Motion consists merely in the fact that... at intermediate times.') then be in the nature of a tautology ? As for Bergson, standing for what has been the people's wont hitherto, he wins our applause, but one wonders what happens to a moving body while in motion, if it is neither in a place nor does it coincide with some "motionless" position. And, one wonders, whether there is just no alternative to having to accept either at-at space-time correspondence as motion or the possibility of moving through space without coinciding with any part thereof. It is contended that, appearances to the contrary notwithstanding, both Russell and Bergson can be seen to be right at the same time, simply by turning their categorical assertions into the consequents of two hypothetical statements. If we add, "If spatio-temporal intervals are (supposed to be) atomistic[4] (or to be more exact, infinzer-atomic[4]) then .....", then Russell's statement is analytically true, and, if we add, "If spatio-temporal intervals are (supposed to be) infinitely divisible[5] (or, to be more exact, imposinfible[6]) then,..." then Bergson's statement is analytically true; and, even though the antecedents are mutually incompatible, the two statements, both being hypothetical, are quite compatible with each other.[7]

I

The controversy concerning the at-at notion of motion, it would seem, stems from the consciously or unconsciously held view that motion is anterior to, or at least co-ordinate with and at the same level as, Space and Time. Now, we know that quite a number of philoscphers have denied the reality of Space and Time, and those who would make Space and Time independent of Motion are but of rare occurence among philosophers. In sooth, the present writer is himself one of those who believe that the physicists' inventory of things that exist would not contain "Space" and "Time". But, this does not mean that 'space' and 'time', as concepts, are necessarily subordinate to and dependent upon the concept of 'motion' in all conceptual systems. What is real may only

be motion and that which is in motion, and in this sense, space and time may not be real. But as soon as motion is desired to be described, space and time present themselves as the handiest conceptual media for the purpose; in fact, we find it not only very convenient but necessary to first postulate a certain constitution of space-time and only then to proceed to enunciate our notion of motion in terms of the hypothesis chosen.

The *postulate* regarding the composition of space/time, in any postulational system, it is contended, is not only independent of but actually more primitive than the *postulate* regarding the specific nature of motion. Once it is realized that the specific mode of motion is a derivative of the nature of space and time, it is quite clearly seen that the question whether immobility set beside immobility can or cannot amount to motion, and whether or not motion consists merely in the fact that a body is in one place at one time and in another place at another time, is really a question whether a given notion of motion is entailed by or compatible with a given assumption regarding the constitution of Space and Time.

It is by no means being implied that I am the only person to have realized that the notion of motion is a derivative or a function of the hypothesis regarding spatio-temporal constitution or that, consequently, assertions regarding motion are only of a conditional nature depending for their validity on the hypothesis chosen : not to talk of such modern writers as Professors Max Black[8], Adolf Grünbaum,[9] G.E.L. Owen,[10] and, V.C. Chappell, many ancient philosophers, certainly Aristotle and Plato, and in all probability Zeno himself, appear to me to be fully cognizant of this fact. What is being suggested is that this fact is not always borne in mind when we discuss problems relating to Space, Time and Motion. Prof. Chappell provides us with a very striking example of what we are contending. In one of the most lucid and illuminating discussions of Space, Time and Motion, "Time and Zeno's Arrow,"[11] Prof. V. C. Chappell until the last but one paragraph of his paper remains fully alive to this phenomenon and clearly implies that the notion of motion is a function of the hypothesis chosen regarding the constitution of space/time but, all of a sudden loses sight of this fact in the concluding paragraph, and obviously succumbing to the charms of the ontological idiom, lapses into the view that

motion could be conceived of independently of spatio-temporal constitution, and goes on to ask,

'What does motion mean or what it ought to mean ? What is motion, really ?'[12]

In what follows we hope to be able to show that each hypothesis regarding the constitution of space/time entails a very different notion of motion, and that as such, no notion of motion can by itself be regarded as right or true, or even as, in Prof. Chappells words, 'the most successful or defensible or adequate;' on the contrary, we can only ask, which of those hypothesis (together with their respective notions of motion) is on the whole the most successful or defensible or adequate. With regard to a notion of motion, we are only entitled to ask whether or not it is consistent with the spatio-temporal hypothesis on which it professes to proceed, and whether or not it has been exhaustively or adequately formulated and, of course, we may ask to what extent it embodies or reflects our respective intuitions of the phenomenon of motion.

If we do not take account of Parmenides' view,[13] or of the hypothesis that spatio-temporal intervals are constituted of infinitesimal parts,[14] we have four distinguishable hypothesis regarding the constitution of Space/Time.

1. Any spatio-temporal interval is divisible *ad infinitum*, but there are points/moments such that a point/moment has position but no magnitude and is contained in spatial/temporal interval, respectively. We shall hereinafter refer to this view as 'the posinfible hypothesis.'
2. Any spatio-temporal interval is divisible *ad infinitum* but no spatial/temporal interval contains anything magnitudeless. This view will be designated 'the imposinfible hypothesis'. (Cf. foot-note 6, *supra.*)
3. Any spatio-temporal interval is constituted of a non-denumerable infinity of magnitudeless points/moments. This will be called 'the infinzer-atomic hypothesis'. (Cf. foot-note 4, *supra.*)
4. Any spatio-temporal interval is composed of a finitude of ultimate constituents, all of unit magnitudes. This will be referred to as 'the finpos-atomic hypothesis.' (Cf. foot-note 4, *supra.*)

As we shall presently see, each of these hypothesis entails a notion of motion, and entails so different a notion from that entailed by the other hypothesis that to the person who subscribes to a given hypothesis all the other notions of motion appear to be simply preposterous. In fact, we shall find that the notion of motion entailed by anyone of these hypotheses cannot even be properly stated if one were proceeding on a different hypothesis.

II

We shall now endeavour to enunciate the notion of motion entailed by each of the hypotheses enumerated above.

(i)

Before we endeavour to see what notion of motion is implied by the hypotheses in question, I should like to present the commonsense view of motion, or the view I believe to be the commonsense view.

In all probability, it is very clearly realized that there is a very intimate relation between Space and Motion as well as between Time and Motion, but there is no realization that Motion can in any sense depend upon the constitution of Space/Time. In fact, we would deem a question regarding the constitution of Space/Time quite irrelevant to the directly observed phenomenon of motion.

I think that we ordinarly suppose that the phenomenon of motion can be analyzed in terms of two inter-dependent phenomena, (i) traversing a distance, and (ii) change of place. This is why we define motion as *passing from one place to another*. I think, we conceive of the relation between motion on the one hand and traversing a distance and change of place on the other hand as even more intimate than that between a whole and its component parts — as there being such a co-implication between the whole (i.e. motion) and the parts (i.e. traversing a distance, and, change of place) that there is a co-implication between the parts themselves. On this view, traversing a distance both presupposes and entails change of place, and change of place likewise both presupposes and entails traversing a distance : If a body is in motion, then it both traverses a distance and changes its locus. If a distance is

traversed, then change of place must occur, and if a distance is not traversed, then there is no change of place; and, likewise, if change of place occurs then a distance is necessarily traversed and if change of place does not occur then no distance can be traversed. To sum it up, traversing a distance and change of place are independent of each other in their conception but necessarily coincident in their occurence.

A body in continuous motion cannot occupy the same space for any period of time however short; but we cannot believe that during its motion the mobile can get out of space altogether. However, we cannot decide whether or not the mobile must always be coincident with a space equal to its dimensions and we do not know what to make of Zeno's claim that at any time during its flight the arrow must be where it is, namely in a space equal to itself, or whether or not to applaud Russell when he affirms that Zeno's claim is a very plain statement of a very elementary fact. We are naturally puzzled by Bergson's statement that a body in motion cannot occupy any space, and believe this statement to tend to irrationality, even though we agree with him that immobility set beside immobility cannot give rise to mobility.

A body cannot properly be said to be in motion if it is not in a state of motion. For thousand of years we have been aware of the phenomenon of relative motion and know that sometimes the arrow remains immobile yet lies many miles away from where it used to be, but, still, we would hold that the arrow does not *move*, it is only *transported* from one place to another. If two trains are running on parallel tracks in the same direction and with the same speed, then relatively to each other they are at rest; but, that is only because they are *moving* in a certain manner. We are startled when Russell tries to make us believe that there could be a movement without the body in question being in a state of motion, as if a person can be said to be angry when he is not in a state of anger.

A body if it is in continuous motion during a period of time t then at any time duringt t it must be in motion, and a body which is at rest at all times during t must be at rest during t.

The world is now very different from what it used to be in an earlier time; even now it is undergoing change: everything is in state of flux and there is nothing in the whole universe which is unchanging. We are ourselves, changing and everything around us is changing. Perhaps, we would hold that the continuous tenses, especilally the Present Continuous Tense, was the best if not the only proper linguistic medium for the enunciation of the fact of motion.

It goes without saying, 'to be at rest' and 'to be in motion' are absolute contradictories. Nothing can ever be both in motion and at rest; anything must at any time be either at rest or in motion. To be motionless is necessarily to be at rest, and not to be at rest is necessarily to be in motion.

(ii)

Before we undertake to determine what notion of motion is entailed by the hypothesis we have christened "imposinfible", I should like to point out that this hypothesis is the purest and the most rigorous form of the hypothesis that Space and Time are divisible *ad infinitum*, and further that unless the postulation of indivisible points of space and moments of time can be shown to be consistent with the supposition of infinite divisibility——a task which has not yet been performed — the imposinfible hypothesis would prove to be identical with the hypothesis of infinite divisibility — the hypothesis on which the greater part of mathematics is based and to which most philosophers subscribe.

Now, if we adopt this hypothesis and proceed to articulate our intuitive apprehension of the phenomenon of motion, we would be led to defining motion in some such way as the following.

Let A be a body, p_1 and p_2 be two such places that each is equal in magnitude to A and there is a distance d between them, and let t be a period of time such that periods of time t_1, t_2 and t_3 together constitute t. Let there also be two places s_1 and s_2 such that each is greater than p_1 and there is a distance d between them. (It is to be recalled that on the present hypothesis we have only divisible intervals of space/time, and that we can have nothing as spatial or temporal which is magnitudeless.) Now, no matter

on what hypothesis we may be proceeding, we would be willing to accept in principle that motion has two components, (i) traversing a distance, and (ii) change of place, or at least that these two phenomena are involved in motion.

On the present hypothesis, traversing a distance would be understood in this way. We would picture to ourselves an arrow entering an empty box (with holes on opposite sides) on one side and coming out of the other, and would refer to this situation by saying that the arrow traverses the length of the box. In an endeavour to be more precise, we would say that if the arrow traverses the box during the period t_2, it is just outside the box (on one side) during some time, say t_1, and again just outside the box (on another side) during another period, say t_3. But the difficulty with this conception would be that alternating periods of rest and motion would have to be postulated if the second component of motion, change of place, is to be specified. For our ordinary purposes it would have sufficed, for, we ordinarily deal with motions which are preceded and followed by periods of rest. But, we do not always deal with motions that alternate with rest; sometimes we have to deal with motions that do not alternate with rest, such as the motions of the planets.

Now, it is to be noted that a body in continuous motion during any period t, cannot on the present hypothesis occupy or coincide with any space during any part of t, for, otherwise it would be at rest during that part of t, and its motion during t would not be continuous. Hence, if we wish to retain the component of change of place, we must so conceive of it that a distance may be traversed during the period it is said to be in that space.

We may, then, define, 'A moves during t' as follows :

(a) A is in p_1/s_1 during t_1 and in ρ_2/s_2 during t_3, and,

(b) A traverses d during t_2.

It is clear that what is really essential is clause (b), or traversing a distance, for, the alternative given in clause (a) is such that clause (a), i.e., change of place, can itself get converted into traversing a distance. Thus, on this view, to move is really to traverse a distance and only involves to be in different (often imprecisely determinable) places.[15]

(iii)

Prima facie, the hypothesis we have called "posinfible" is self-contradictory, and. as such, no hypothesis. On the one hand the assumption is that any spatio-temporal interval is divisible and on the other hand we have postulated indivisible points and moments as contained in them. It would appear that in an endeavour to have the good points of both the infinitely divisible and the atomistic hypotheses, we have arrived at a combination which is logically impermissible. Although a great majority of philosophers and mathematicians from Aristotle's day to our own have consciously or unconsciously adopted this hypothesis, no satisfactory account of this hypothesis has yet been given. If Russell would have been right in believing that Whitehead's method of extensive abstraction gives a complete and satisfactory account of the procedure of arriving at points and moments, then a successful defence of posinfibility would have been possible. But Professor Grünbaum has shown (i) that Whitehead's method is defenceless against Zeno's argument that no set of magnitudeless points can give rise to a finite interval of space, since this method does not provide for a non-denumerably infinite set of points constituting an interval and no merely denumerable set of points can give rise to a positive interval, and (ii) that the convergence of the abstractive classes is fatally ambiguous in as much as two distinct but neighbouring poins, such as $x = 0$, and $x = 10^{100\upsilon}$ though separated by an infinity of points, cannot, by sense perception, be unmistakeably distinguished.[16] I must confess that I have not yet had the opportunity to study this method with the care it deserves.[17] But it seems to me that Professor Grünbaum's objections are valid and that defenders of Whitehead's method have not really succeeded in meeting Professor Grünbaum's objections.[18] Furthermore, it seems to me that there is another objection to this method. It is not very clear from what Whitehead says whether a given abstractive class (or a set of abstractive classes) is itself to be regarded as identical with the point to which the abstractive class or classes would ordinarily be said to converge, or whether it is the 'logical sum or product' (i.e. something common to every element) of a given abstractive class (or of every member of a given set of abstractive classes) which is to be identified with a point.[19] But, in either case, Whitehead's method is a failure. If a point is defined as the logical product of an abstractive class then the obvious objection is that an abstractive class may not have

a logical product, rather, it cannot have a 'logical product', for, by definition, no member of the class is included in all the other members. Since Whitehead begins with the assumption that points must be contained in those areas which converge to it, he simply takes it for granted that an abstractive class must have a logical product and that this product must be the point in question. But I do not see how can any set of areas be said to necessarily contain a given point without assuming that which is to be proved, namely, that (i) *there are* things called 'points', and that (ii) points *are* contained in areas. On the other hand, if a point is defined as an abstractive class, I for one fail to see how a body can be said to lie between, or to traverse the distance between, points a and b, if to do so the body must lie between or travers the distance between the abstractive sets A and B (converging towards a and b respectively).

For many years now I have been endeavouring to work out a satisfactory solution of this problem, and I have often felt to be on the verge of obtaining a definition of 'point' in keeping with the assumption of infinite divisibility, only to find out a hidden circularity. However, as far as the moment is concerned, I believe that it can quite easily be defined without involving a circularity or inconsistency with the hypothesis of infinite divisibility. This is so because the temporal extension is one dimensional, and a unique moment is defined by dividing Time into 'time past' and 'time future', but, no similar operation can be performed on a three dimensional quantity. However, I have not yet despaired of finding a way out, and the obvious advantages that this hypothesis offers encourage me to believe that, sooner or later, a successful defence would become possible.

With regard to the notion of motion entailed by the posinfible hypothesis, the first point to be noticed is that although it is a very complicated matter to show that a body coincides with a space equal to its dimensions at some moment during its motion, it can be shown that there is a sense in which a body in motion occupies a space at any moment.[20]

The second point to be noticed is that but for the above difference, the notions of motion entailed by the imposinfible and the

posinfible hypotheses are the same. Thus, 'to move' may on the present hypothesis be defined as follows (it being given that m_1 is the end-point common to periods t_1 and t_2, and m_3 is the end-point common to t_2 and t_3) :

(i) A is in p_1 at m_1 and in p_2 at m_2, and
(ii) A traverses d during t_2

Now, the relation between these two components is essentially the same on this hypothesis as it is on the imposinfible hypothesis except that although clause (i) is entailed by clause (ii), it is not reducible to clause (ii); in fact, clause (i) provides precision to clause (ii). Thus clauses (i) and (ii) are really independent in their conception but concomitant in occurence.

It may also be mentioned that no attempt to reduce clause (ii) to clause (i) can be successful either.

(iv)

On the infinzer-atomic hypothesis, Space/Time are conceived of as having ultimate, that is, indivisible, constituents of zer magnitudes, the so-called degenerate intervals (or in ordinary parlance, points and moments) and hence that any positive interva of Space/Time is constituted of a non-denumerable infinity of degenerate spatial/temporal sets, i.e. of points and moments. Since moments (or degenarate moment-sets) are magnitudeless, no distance can be traversed in a moment, for, distances being positive intervals of space, and, as such, being necessarily divisible, if a distance could be traversed in a moment then moments would have to be necessarily divisible and hence magnitudinous. But nothing can at any moment occupy a greater space than its dimensions. Hence, at any moment, a body must occupy a space equal to itself.

Consequently, if motion cannot take place in a moment then motion must consist merely in being in different places at different times. Now, on the present hypothesis, no two moments or points are consecutive, there being in fact a non-denumerable infinity of moments and points between any two moments/points. Hence, between any two positions occupied at two given moments, there must lie another position which is occupied at a moment lying between the two given moments. Thus, on the infinzeratomic hypothesis, 'to be in motion' *is* 'to be in different places at different

moments and in intermediate places at intermediate moments.' We shall refer to this as the 'infinzer atomic' or the 'at.....at.....at' notion of motion.

(v)

On the hypothesis of finpos-atomicity, there are ultimate components of Space/Time, indivisible but of unit-magnitudes. These ultimate constituents of Space/Time are so different from the points/moments of the hypotheses of posinfibility and infinzer-atomicity that we would do well to use different words for them. We shall therefore refer to the ultimate constituents of Space and Time as the 'microtopes' and 'microchrones' respectively.

Ordinarily we restrict the word 'during' for use in relation to periods of time, using the word 'at' in relation to moments, instants and the like. But a microchrone, though indivisible, is a component of time; hence it should be regarded more as a 'period of time' of the other hypotheses and of the commonsense view than as their 'moment'. We shall therefore use the word 'during' in relation to microchrones to emphasize that these are component parts of time such that a (finite) collection of them is identical with any given temporal interval.

Now, unless we can assume either that a body of the size of a microtope can during any microchrone occupy more or less space than a microtope or that while in motion it does not occupy any space at all, it would follow that the body must during each microchrone occupy a space equal to itself, that is, be in a microtope. During no microchrone can the arrow be in two or more microtopes (since it is of the size of one microtope only, and the microchrone, being indivisible, does not permit of any processes), or be in a place smaller than a microtope (since microtopes are indivisible, as also because it cannot occupy a place smaller than itself); hence it would either occupy a microtope or occupy no space at all. But the question is, what would it mean to occupy no space at all ?. It can only mean to lie between two microtopes or not to exist at all. But, on the present hypothesis, any (finite) interval of space is exhaustively given by a (finite) collection of microtopes; hence, unless it means not to exist at all, to occupy no space can mean just nothing. But, a body must exist in order to be able to

be in motion; hence, the arrow must occupy a space equal to itself during each microchrone of its flight.

Now, during no microchrone can anything pass from one place to another or traverse a distance, for, on the present hypothesis, 'to traverse a distance' and 'to pass from one place to another' can only mean to be in two or more microtopes or collections of microtopes, and this will make the microchrone divisible. In fact we would have to define all such notions in terms of being in or occupying microtopes. In other words, the only manner in which a body of the size of a microtope can accomplish motion on the present hypothesis is by being in one microtope during one microchrone and in another microtope during some other microchrone. Thus, on this hypothesis, 'to move' is 'to be in one microtope during a given microchrone and to be in another microtope during another microchrone.' We shall designate this 'the at-then-at notion of motion'. Not only on the commonsense view but also on most other hypotheses, to occupy a (proper) place during any given period is to be at rest during that period, and even on the present hypothesis we must regard the body as at rest *in relation* to the given microchrone if a body occupies a given microtope during that microchrone[21] and we have seen that that is what invariably happens; hence, we may playfully characterize finpos-atomic motion as 'restful'.

We would further find that a body while in continuous motion may remain in the same place (a microtope, or a collection of microtopes) for very many microchrones, i.e. there may be a composite sub-period within that period of a body's motion during the whole of which the body may remain in the same place,[22] and may therefore be said to be undoubtedly at rest during that sub-period. Such a sub-period would be a period on any possible definition of being a period, and any possible definition of being at rest would be applicable to the body in question. Moreover the speed of a body in motion can be described only as the rate of *remaining in*, i.e., continuing to occupy or be in, *the same place*. Hence, the finpos-atomic motion may, again playfully, be characterized as 'truly restful.'[23]

Another feature is what may be said to be 'passing without encountering'. When two parallel rows of bodies move in oppo-

site directions, and 'to encounter' is understood to mean 'to be in the same column at the same time (i.e. at the same microchrone),' then every body of one of the two rows may not encounter each body of the other row.[24]

III

We have seen that the notions of motion entailed by the two species of the hypothesis of infinite divisibility, the posinfible and the imposinfible hypotheses, are very close to the commonsense notion and that the two atomistic hypotheses, the infinzer-atomic and the finposatomic, entail notions of motion which are considerably removed from the commonsense view of motion.

However, it would not be quite correct to think that the notion of motion of either the imposinfible or the posinfible system embodies or corresponds to the commonsense notion. Each differs from the commonsense notion in its own way.

The imposinfible hypothesis entails a notion which agrees with the commonsense view in regarding motion and rest as involving absolutely contradictory phenomena, in not allowing a period of rest within a peiod of continuous motion, and in regarding the continuous tenses as the only appropriate modes of expressing motion. But it is at variance with the commonsense notion in regarding a body as not being in any space while in motion.

The posinfible notion of motion, while it does away with the difficulty of not occupying a (proper) place during motion (by distinguishing between 'occupation during' and 'occupation at'), it transgresses the commonsense view by being obliged to regard motion and rest as contradictory only in relation to periods, for, various considerations necessitate that a body should be regarded as being neither in motion nor at rest in relation to moments of time.

The infinzer-atomic notion of motion is farther removed from the commonsense notion than the imposinfible and the posinfible notions. Against this view two objections have been raised : firstly, that it fails to preserve the continuity of motion, and,

secondly, that it involves a miraculous change of place. We shall consider these objections in some detail.

Bergson argued that while in flight if the arrow ever occupied a space or coincided with anything immobile, its flight would not be continuous, and the arrow would be engaged in a series of runs and stoppages. Now, since Russell rightly regarded a body as at rest at any moment, we may re-frame Bergson's objection thus : a body in motion during the period $m_1 - m_2$ cannot be in continous motion since there are constituents of $m_1 - m_2$ at which the body is at rest. The answer to this objection would be that the conception of 'continuity of motion' from which this objection arises is alien to the infinzer-atomic system, and that a certain mode of motion would be continuous in a sense which was parallel to the ordinary sense of continuity of motion. On the present hypothesis, a body would be in continuous motion if it successively occupies all the points between the end-points (i.e. does not skip over any intermediate point) and if it does not occupy any point for more than one moment.

The other objection may be presented in the words of Professor Lazerowitz :

> It is the same with the claim that the motion of a body consists merely in the body's occupying a series of different places, without passing from any one place to any other. Whatever it is he is doing, a philosopher who says this cannot be so unusual, and commit such outrage against our intelligence, as to be declaring that the word motion *means the same* as the phrase 'succession of static states of rest.'[25]

I take it that Professor Lazerowitz means that if the arrow is ever (i.e. at any moment) at rest and never (i.e. at no moment) traverses any distance, then no change of place can occur; to assume that a change of place occurs would in these circumstances be quite absurd. Russell held that such a censure would be valid if the 'succession of static states of rest' would have been consecutive; but on his (i.e. infinzer-atomic) view, no such absurdity was involved. He thought that in virtue of there being an infinitude of positions between any two positions, a body does not just miraculously

disappear from a given position and re-arise in another position, it reaches the latter position by traversing the intervening distance. Although Bergson may not himself have intended to say so, we may here present his argument as the claim that the infinity (even non-denumerable infinity) of intermediate positions notwithstanding, the body traverses no distance, it merely (and Bergson would have certainly felt, miraculously) occupies different positions. It is clear that Bergson would have been right and Russell would have been in the wrong : on the infinzer-atomic hypothesis no distance can be traversed *in contradistinction to* being in different places at different times. But, the point is, this does not show that a succession of static states of rest is not motion, it only shows that 'to traverse a distance' cannot mean anything other than, and must itself be defined in terms of, 'being in different places', or, alternatively, that if to traverse a distance is something other than, and over and above, being in different places at different times, then the expression "to traverse a distance" mean nothing on the present hypothesis. Thus, if 'to traverse a distance'means something in itself, something of which we are aware only sensibly or intuitively, and if what it means is not comprehended within the notion of 'being in different places' then Russell was certainly wrong in believing that mere occupation of different places could constitute motion. But, the fact is that, on the present hypothesis, Russell is right in his definition of motion, and wrong only in believing that his definition permitted anything which could be said to be 'traversing a distance' such that it was not analyzable in terms of 'being in different places.'

The same charge of miraculous change of place can be levelled against the at.....at.....at notion of motion from the finpos-atomic point of view. How can a body go on to occupy a point an infinitude of points away without first occupying the next point ? Russell gave a very correct answer to this question: there being no next point (on the infinizer-atomic hypothesis), no question of going to a next point arose. In other words, what would be necessary on another hypothesis, and which is alien to the present hypothesis, is being illogically imported into a discussion proceeding on this hypothesis.

Another objection to the infinzer-atomic notion of motion

could be that it presents the *result of motion* as *motion*. But this would be tantamount to adopting the imposinfible or the posinfible hypothesis. It is only on the posinfible hypothesis that a position comes necessarily to be occupied and on the imposinfible hypothesis may come to be occupied *as a result of* traversing a distance. On the present hypothesis, no distance is traversed resulting in the occupation of a position; on the contrary, a distance could (by convention) be said to have been traversed as a result of the occupation of different positions. To reiterate what has already been said, there can be no such thing as 'passing from one place to another' or 'traversing a distance' if these themselves do not mean 'being in different places at different times and in intermediate places at intermediate times' or are not otherwise derived therefrom.

All that needs to be said in defence of the 'at.....at.....at' notion of motion is that different notions of motion involve different conceptual systems and that the infinzer-atomic notion is so systematically connected with the other notions of motion that it performs in its system the same function as is performed by the other notions of motion in their respective systems, namely, articulation of our intuitive apprehension of the phenomenon of motion in keeping with the dictates of the conceptual system chosen.

The finpos-atomic notion is the farthest removed from the commonsense notion and has consequently come under attack from all quarters, even from Russell himself.

On the finpos-atomic hypothesis, motion consists in being indifferent microtopes (or collections of microtopes) during different moments. But, this appears to involve a miraculous change of place. How can a body which is at rest in microtope p_1 during microchrone m_1 manage to pass on to an adjacent microtope p_2 during microchrone m_2, or for the matter of that, any other microchrone ? But this difficulty is no real difficulty. When we say that the change of place would be miraculous, what we are really doing is wondering as to how could a change of place come about if no distance is traversed. But, on the present hypothesis, 'to traverse a distance', as on the infinzer-atomic hypothesis, means nothing if it does not itself mean being in different places during

different microchrones. To demand that a body should effect change of place only in virtue of traversing a distance would be to adopt the imposinifible hypothesis, and to demand that a body should effect change of place only concomitantly with traversing a distance would be to adopt the posinfible hypothesis.

We have said earlier that the finpos-atomic hypothesis entails that there should be (composite) sub-periods of rest during periods of continuous motion. On all other views this would appear to be quite absurd : it is precisely motion of this nature which would be said to be 'discontinuous'. Now, on the finpos-atomic hypothesis, we can distinguish between 'absolutely continuous' and 'relatively continuous' motion : if a body is in a different place every other microchrone, then its motion is absolutely continuous, and if it is in a different place every n-th microchrone then its motion is 'relatively continuous'. Now, with regard to our feeling that what we have called 'relatively continuous' is really discontinuous, I would urge that we would again be importing a notion derived from a contrary hypothesis. On the present hypothesis, the concept of 'relatively continuous' performs the same function which the concept of 'continuous' performs in the other conceptual systems, namely, expressing our intuitive apprehension of the phenomenon we ordinarily call 'continuous motion.'

The phenomenon we have referred to as 'passing without encountering', as even Professor Grünbaum clearly states, only shows that what would be an event on one hypothesis is a non-event on the present hypothesis.[26] In other words, while passing without encountering is impossible in the posinfible and the infinzer-atomic systems, and it is necessary that each body of one row must encounter each body of the other row, on the hypothesis of finpos-atomicity, there would be cases where such an encounter would not take place. Now, on the imposinfible hypothesis also, no such encounter takes place : actually, no encounter can at all take place, for, no body of either row would be occupying any proper place during any part of its period of motion. Now, the point is that the feeling that every body of one row should encounter every body of the other row stems from the assumption of posinfibility/infinzer-atomicity of spatio-temporal intervals, and once this is realized, the present problem is seen to be no problem.

IV

We have seen that each hypothesis regarding the constitution of Space/Time entails a notion of motion, and, as we said earlier, entails so different a notion of motion that to a person who adopts any one of these hypotheses all other notions of motion than the one entailed by the chosen hypothesis appear to involve something miraculous or impossible or absurd. However we have seen that no notion of motion can as such be regarded as truly representing the sensible phenomenon we call motion or even as being the more adequate or successful or defensible. What is or is not adequate or satisfactory are in a very fundamental sense only the hypotheses regarding spatio-temporal constitution and derivatively the conceptual systems developed on the basis of those hypotheses including their respective notions of motion.

It goes without saying we would be pre-disposed in favour of that notion of motion which appears to us to answer to our intuitive apprehension of motion, and as such we would incline towards that hypothesis which entails that notion of motion. Even so, we cannot on that account alone regard anyone of these hypotheses as the only possible, or even as the most satisfactory one. Before we accept any conceptual system we have to consider all its other aspects and only then can we decide which of the hypothesis is on the whole the most satisfactory.

The infinzer-atomic hypothesis can easily be said to be the modern mathematico-scientific view of Space/Time, for, this is the hypothesis accepted by a great majority of contemporary mathematicians and philosophers of science and which Einstein's theory is said to imply. To me however this hypothesis appears to involve a self-contradiction, as we have endeavoured to show in an earlier paper.[27]

We have pointed out above that despite the fact that the posinfible hypothesis offers numerous advantages it has not yet been shown that its two fundemental components, (i) spatio-temporal intervals being infinitely divisible, and (ii) there being indivisible points and moments, are consistent with each other. If so, to base ourselves on this hypothesis would be to base ourselves on very insecure foundations.

Thus, we may be left with the imposinfible and the finpos-atomic hypotheses as the only viable hypotheses. In that case, there will be no alternative to being in motion but not 'in space' or being in different places but not in a 'a state of motion'. Aristotle had thought that a continuous magnitude could not be finpos-atomic, and, many recent writers have held that Zeno's paradoxes are valid arguments against the assumption of finpos atomicity. If this would have been true, then the imposinfible hypothesis would alone have remained in the contest. But, as we have argued else where,[25] this is not so : no matter whether we like to proceed this hypothesis or not, all arguments urged against the finpos-atomic hypothesis are circular or involve some other fallacy.

To me personally, the posinfible hypothesis — once the question of inconsistency has been settled — appears to be the most satisfactory of all these hypotheses for purposes of Science and Mathematics. In fact, I believe that by far the greater part of mathematics is founded on the posinfible hypothesis, and that the real achievements made in mathematics on the basis of the infinzer-atomic hypothesis can be evolved also on the basis of the posinfible hypothesis.

To conclude our discussion, it seems to me that the task ahead lies in two directions. Firstly, effort is to be made to solve the fundamental problem of the posinfible hypothesis, namely, to evolve points and moments by proceeding on the hypothesis that spatio-temporal intervals are divisible *ad infinitum*. And secondly, every effort is to be made to retain the atomistic, or to be more precise, the finpos-atomic, foundations of Arithmetic and to evolve a viable Geometry and Dynamics on the basis of the finpos-atomic hypothesis.

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NOTES

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1. Bertrand Russell, *Mysticism and Logic*, Hearmondsworth, 1918, reprint 1954, pp. 82-83.

2. H. Bergson, *Creative Evolution* (Tr. A. Mitchell), New York, 1911, reprint 1944, p. 331.
3. Ibid, p. 335.
4. An interval of Space/Time (i. e. Space and/or Time, or Space-Time) shall be said to be *atomistic* if it is supposedly composed or constituted of indivisibles, irrespective of whether these indivisibles are assumed to be of zero magnitude or of positive, i.e. unit, magniude. We shall however refer to an interval as infinzer-atomic/finpos-atomic when the nature of the indivisibles is desired to be specified, *infinzer-atomic* if it is supposed that (i) the indivisibles have zero magnitude, and (ii) finite intervals are constituted of a non-denumerable infinity of them, and, *finpos-atomic* if it is supposed that (i) the indivisibles are of unit magnitude, and (ii) finite intervals are constituted of a finite number of them.
5. The hypothesis that for any value of x, if x is a spatial/temporal interval then x is divisible.
6. The hypothesis that (i) for any value of x, if x is a part of Space/Time, then x is divisible, and (ii) for any value of x, if x is a spatial/temporal term, then there is a y such that y is a part of Space/Time and y is the referend of x. (That is to say, there is nothing spatial/temporal other than the parts of Space/Time, and that any part of Space/Time is divisible. Thus, there are no points and moments, and the words 'point' and 'moment' are not spatial/temporal terms, and, as such, are meaningless, if these do not happen to be meaningful non-spatial/non-temporal terms; or, in other words, indivisible points/moments, even as magnitudeless limits of spatio-temporal intervals, are not to be postulated).
7. As it is,the two propositions, having the form of 'If A then B' and 'If not A then not B', are complementaries, and are of such a nature that it is quite tempting to believe that the one is inferable from the other.
8. *Problems of Analysis*, London, 1954.
9. *Modern Science and Zeno's Paradoxes*, London, 1968.
10. "Zeno and the Mathematicians" *Proceedings of the Aristotelian Society*, LVIII (1957-58), pp. 199-222.
11. *The Journal of Philosophy*, LIX (1962-63), pp. 197-213.
12. Ibid., p. 213.
13. Parmenides, as is well known, regarded the whole universe as an indivisible unity in which, apparently, there could be no distinguishable parts. We have discussed this view in "Zeno's Paradoxes : Towards a Solution at Last", *Islamic Studies*, XI (1972), pp. 125-151.
14. Not only have mathematicians discarded this hypothesis, the infinitesimal hypothesis is founded on the fallacy of appearing to define an actual entity (viz., the class of infinitesimals) while in fact only defining a hypothetical relation between an actual and a hypothetical entity. If Y is of positive magnitude, then there is no X such that X is infinitesimal with respect to Y, though we do know what it would be like for X to be infinitesimal with respect to Y if there were such an X. We have therefore not considered it necessary to discuss the notion of motion entailed by this hypotheiss.

15. It may be pointed out that imprecision is inevitable on the imposinfible hypothesis not only in respect of motion but even in respect of the intervals of space and time themselves. (See David Hume, *A treatise of Human Nature*, reprint London, 1951, vol. I, pp. 51-59; we have dealt with this question at some length in *Towards a Definitive Solution of Zeno's Paradoxes*, Karachi, 1973, pp. 19-38.

16. A. Grunbaum, "Whitehead's Method of Extensive Abstraction", *The British Journal for the Philosophy of Science*, IV (1953), pp. 215-226.

17. Whitehead has presented his method in (*a*) three of his books, (i) *An Enquiry Concerning the Principles of Natural Knowledge* Cambridge, 1919, Part 3 (i) *The Concept of Nature*, Cambridge, 1926, chapter 4, and (iii) *Process and Reality*, New York, 1929, Part 4, and in (*b*) his paper, "La theorie relationiste de l'Espace", *Revue de metaphysique et de morale*, XXIII (1916), pp. 423-454. As a very sympathetic account of Whitehead's method, C. D. Broads *Scientific Thought* (London, 1952, pp. 33-56, especially pp. 44-52) is very helpful in understanding this method.

18 For example, C. D. Broad (op. cit.), L. S. Stebbing (*A Modern Introduction to Logic*, London, 1958, pp. 446-452), and W. Mays (*The Philosophy of Whitehead*, New York, 1962), defend Whitehead's method. Professor Mays has devoted a chapter (pp. 115-125) to a consideration of Professor Grunbaum's objections.

19. Thus, even Broad is clearly inconsistent on this point. On p. 44 (op. cit.), he maintains that points cannot be defined as the limits of a series of smaller and smaller volumes, one inside the other, and 'boldly' goes on to define points as such series themselves. On p. 45, in view of the fact that many such series may converge to the same point, he defines a point as 'the class of all volumes in any of the series which would commonly be said to converge to the point'. But on p. 51, after stating that the points 'are classes of series of volumes", Broad adds, 'or to be more accurate, are the logical sums of such classes."

20. Just as having a point is a very complicated matter on the posinfible hypothesis, occupying a space at a moment is likewise a very complicated matter. Prof. Max Black has endeavoured to define momentary occupation of a position in terms of a body's volume and the distances traversed during a period ending in the given moment and a period beginning with that moment (*Problems of Analysis*, pp. 140-144). To me it seems that this definition involves a circularity, for, while the 'space reserved' during a period p, (containing moment m) as well as the 'space reserved' during period p_1 (a part of p terminating in moment m) can be spelled out if the body is at rest during the period following p or p_1 (as the case may be), it cannot be determined what space is reserved during p_1 if the body continutes to be in motion during p_2 the sub-period of p following p_1. (By 'space reserved', Professor Black means the volume of the body plus the proper volume through which the body passes during the period in question.) However, we believe that we can define 'reaching a position x at moment m'in terms of the distance traversed during p_1 and that the distance

traversed during p_1 can be determined without any illicit recourse to position x or moment m.

21. We have discussed this point in an earlier work, "Motion in an Atomistic Space/Time,' (*History of Science in Central Asia, Proceedings of the National Seminar on History of Science in Central Asian Civilisations*, Islamabad, 1978, pp. 33-60; see, pp. 45-47.
22. That this must be so was known to some medieval Muslim Atomists; see; Fakhr al-Din al-Rāzi, *Kitāb al-Māba ith al-Mashriqiyah*, Hyderabad, Deccan, 1343. A.H. pp. 602-604, and Moses Maimonides, *The Guide for the Perplexed* (tr. M. Friedlander), reprint, London, 1956, pp. 121-122. Bertrand Russell (*Our Knowledge of the External World*, reprint, Woking and London, 1961, p. 183), rightly argues that one-one microchrone-microtope correspondence is the fastest possible motion, and that "any slower motion must be one which has intervals of rest interspersed". We have discussed this point at some length in "The Atomistic Hypothesis Reconsidered", *The Pakistan Philosophical Journal*, vol. XIII, no. 2 (Jan-June 1975), pp. 14-42; see, pp. 20-21.
23. Since to move is to be in different microtopes at different microchrones, and a body may remain in the same microtope for very many microchrones, the only fundamental way of describing a body's speed on the present hypothesis would be to state for how many microchrones does it remain in each microtope or each collection of microtopes. Derivatively, of course, speed may be defined in terms of number of microtopes visited during a number of microchrones, or, in other words, 'distance traversed' during 'periods' of time.
24. See, e.g., G. Noel "Le mouvement et les arguments de Zenon d' Elee" *Revue de metaphysique at de morale*, vol. I (1893), pp. 107-125, who however assumes that one body of one of the moving rows must pass two bodies of the other moving row and hence be divisible (p. 116); and, F. Evellin, "Le mouvement et les partisans des indivisibles", *Revue de metaphysique et de morale*, vol. I (1893), pp. 382-395, who argues that the phenomenon in question does not involve division of the supposed indivisibles but only passing without encountering, which is entailed by the hypothesis in question (p. 386).
25. M. Lazerowitz, "The Paradoxes of Motion", *Proceedings of the Aristotelian Society*, LXX (1951512), pp. 26-280; see, p. 270.
26. A. Grūnbaum, *Modern Science and Zeno's Paradoxes*, pp. 117-120.
27. "Infinzer-atomicity", *Pakistan Philosophical Journal*, XIII, No. 3 (October, 1975), pp. 47-84, and XIV, No. 2 (Jan-June 1976), pp. 4-72.
28. 'Zeno's Paradoxes : A Solution Hazarded" (*Procedings of the Pakistan Philosophical Congress*, XV, 1968, pp. 143-156), and, "The Atomistic Hypothesis Reconsidered" (*The Pakistan Philosophical Journal*, XIII, No. 2, Jan-June 1975, pp. 14-42).