

ASSERTION IN FREGE AND WITTGENSTEIN'S TRACTATUS¹

I propose to discuss two men, Gottlob Frege and Ludwig Wittgenstein, the two languages they devised, and by implication, at least, the two different cultures they seem to have represented. Even this narrowing of the session's topic, "Language, Culture, and Man", leaves me with material too vast to deal with in the limited time allotted. So I shall focus on one small area of their difference. The issue is whether a special assertion sign or "judgment stroke" is necessary to distinguish an asserted sentence from one that is merely propounded for consideration.

Modern symbolic logic dates from the publication in 1879 of Frege's *Begriffsschrift*. Eighteen years later, in his essay "On Herr Peano's *Begriffsschrift* and My Own," Frege explained the motivation for his work in these terms: "I became aware of the need for a *Begriffsschrift* when I was looking for the fundamental principles or axioms upon which the whole of mathematics rests." (III, 1)². Frege then described briefly the need for a special symbolic language, or "concept writing", in order to make all assumptions or premisses explicit, and also to make explicit the modes or forms of inference used in moving from premisses to conclusions.

Until a special artificial symbolic language has been constructed and adopted for use in a scientific community, one is forced to use ordinary language, natural language. But Frege continually warns us against this or that misleading aspect of ordinary language, even as he uses it, writing: "So a battle with language takes place and I am compelled to occupy myself with language although it is not my proper concern here." (VII, 519). Frege blamed previous lack of progress in studying the foundations of mathematics on "... the logical imperfection of our languages" (III, 1), going on to explain that "The reason why verbal languages are ill-suited to this purpose lies not just in the occasional ambiguity of expressions, but above all in the absence of fixed forms for inferring." (III, 2).

Especially important among Frege's contributions to the development of symbolic logic was his invention (or discovery) of a theory and notation for quantifiers, propositional functions, and variables, far superior to anything previously devised. Dummett called it "the deepest single technical advance ever made in logic." (II, xv). The earlier insight of Kant that "existence is not a real predicate" is captured and built into Frege's notation. There the proposition affirming the existence of God is symbolized by the application of the existential quantifier to the concept "x is Divine" rather than by predicating existence of the Deity Himself. As Frege wrote: "Because existence is a property of concepts the ontological argument for the existence of God breaks down." (V, 65). The power of the new notation is best revealed in connection with multiply general propositions, where the order of quantification can be of decisive importance. Various fallacies are due to confusion about order of generalization. For example, Aristotle argued early in his *Ethics* that since every action is taken for the sake of some end, therefore, there is some end for the sake of which every action is taken. That argument has only to be symbolized using multiply general quantification for its error to be exposed. The case is the same for the variant of the "first cause" argument which proceeds from "Everything has a cause" to "There is a Cause of all things." (cf. VI, 1-13). Contrasting ordinary language's method of expressing quantification with the more precise method of his *Begriffsschrift*, Frege inveighed against "constructions in which language seems to have indulged in order to mislead logicians." (III, 5, see also IV, 13-14)

There is a tension, then, in Frege's *using* ordinary language in the process of *constructing* his *Begriffsschrift*, which he refers to as "a logically perfect language." (IV, 70). The cause of that tension was recognized by Frege to be the fact that "... languages are unreliable on logical questions." (IV, 126)

Frege was aware, of course, that natural languages have uses other than the logical or scientific. Especially in his essay "On Sense and Reference", Frege acknowledged the legitimacy and importance of these other uses, writing: "In hearing an epic poem, for instance, apart from the euphony of the language we are interested only in the sense of the sentences, and the

images and feelings thereby aroused. The question of truth would cause us to abandon aesthetic delight for an attitude of scientific investigation." (IV, 63). Of course the "...shading, or colouring of the thought...is irrelevant for logic." (IV, 40n). Frege wisely remarks that: "...what is essential depends on one's purpose. To a mind concerned with what is beautiful in language what is indifferent to the logician can appear as just what is important." (VII, 516). Frege notes that the "truth claim" that typically arises "from the form of the declarative sentence" does not do so when the latter "...lacks its usual force, e.g. in the mouth of an actor upon the stage..." (IV, 64). Ordinary language and Frege's extraordinary symbolic language were different, then, in that the former has important poetic, dramatic, and sentimental uses not shared by the latter.

Instead of analysing propositions into 'subject' and 'predicate' as the traditional logic would do, Frege followed the mathematical practice of discussing *functions*, *arguments of functions*, and *values of functions for given arguments*. Many different varieties of functions are admitted: *arithmetic*, *equational*, *descriptive*, and *propositional*.

Still another group of functions emerges when compound sentences of the sort we call "truth functional" are analysed. These are the truth functions expressed by "if () then ()," "() or ()," "not ()," plus more complex truth functions by combining these. Sentences are the argument expressions inserted into the blanks of these truth function expressions, and the values of these functions for given arguments are truth values, which are expressed by the compound sentences formed by the insertion of sentences into the gaps of the truth function expressions.

It is obvious that a compound sentence like a conditional (or a disjunction or a negation) can be asserted without its component sentences or subordinate clauses being themselves asserted. Clearly, then, a sentence, even one declarative in form, can be uttered without being asserted. Frege may have been influenced by this consideration to introduce the special assertion sign, or "judgment stroke", " \vdash ", which when prefixed to a declarative sentence indicates that the sentence is asserted rather than simply uttered to express "a mere supposition". (IV, 34)

In ordinary language there are many things done in addition to making assertions. Sometimes differences in the kinds of speech acts performed are signalled by punctuation marks, “?” and “!”. Frege may have been influenced by these considerations in introducing the special assertion sign “|-”. Had he been interested in all kinds of communication instead of just the scientific, assertoric kind, he might well have introduced a whole arsenal of prefixes appropriate to the interrogative, imperative, operative, etc., as well as the assertive.

Frege was well aware of the importance of context, on what speech act, if any, an utterance in natural language performs. Context is not required merely to resolve the ambiguity of utterances containing indexical or token-reflexive terms. Context and intention determine whether an utterance is an assertion. One of Frege's repeated examples shows this: “If an actor utters a statement on the stage, it cannot be said that he *really* asserts anything. . . .” (VII, 569). If the formulae of the *Begriffsschrift* are to be independent of context in the way that expressions in ordinary language are not, something is needed to distinguish asserted from non-asserted formulae in it. Frege may have been influenced by these considerations in introducing the special assertion sign “|-”.

Let us turn now to Wittgenstein's *Tractatus*, a difficult and fascinating book of wide-ranging influence, which has been said to be “virtually unintelligible without an understanding of its Fregean background” (II, 662). Wittgenstein's *Tractatus* presents a world composed of facts, (IX, 1.1) where a fact is the existence (IX, 2) or non-existence (IX, 2.06) of an atomic state of affairs,³ which is a combination or connection (IX, 2.01) or configuration (IX, 2.0272) of objects, each of which is simple (IX, 2.02). The existence of an atomic state of affairs is asserted by an elementary sentence (IX, 4.21) which consists of names (IX, 4.22) in immediate combination (IX, 4.221). These names are simple symbols (IX, 4.24) each of which means an object (IX, 3.203). And the picture theory has it that: “The configuration of objects in a situation corresponds to the configuration of simple signs in the propositional sign”, (IX, 3.21) which is “a picture of reality” (IX, 4.01).

Wittgenstein rejects the Fregean assertion sign, writing: “(Frege’s ‘judgment-stroke’, ‘ \vdash ’ is logically quite meaningless: in the works of Frege (and Russell) it simply indicates that these authors hold the propositions marked with this sign to be true. . . .)” (IX, 4.442). Because Frege does not tell us explicitly why he introduced the judgment-stroke or assertion sign into his *Begriffsschrift*, we cannot be sure exactly what parts of Wittgenstein’s *Tractatus* are supposed to negate Frege’s reasons for introducing it. We have conjectured several kinds of considerations that may have influenced Frege in this connection. A careful reading of the *Tractatus* shows that Wittgenstein rejected all of them.

Although Frege was not altogether clear about the matter, it seems that he was inclined to apply his *Begriffsschrift* analysis to ordinary language. Even though all of his formulae that are not components of other formulae, in both his *Begriffsschrift* and his *Grundgesetze der Arithmetik*, are assertions, Frege was cognizant of many non-assertoric uses of ordinary language. So Frege may have believed that the assertion sign was needed to distinguish assertions from non-assertions.

But Wittgenstein in the *Tractatus* limited *all* language to the scientific, assertoric use. Only “propositions of natural science” can be said (IX, 6.53): “. . . it is impossible for there to be propositions of ethics”, (IX, 6.42) or aesthetics (IX, 6.421). “Propositions can express nothing of what is higher.” (IX, 6.42) For Wittgenstein each elementary proposition is an assertion (IX, 4.21), every “proposition is a truth function of elementary propositions” (IX, 5) and “The totality of propositions is language.” (IX, 4.001) So for Wittgenstein there was no need for an assertion sign to mark off assertions from non-assertions, because there are *only* assertions. It is not that all propositions are asserted, but rather that every utterance is assertive.

We remarked that for Frege context was often decisive in distinguishing assertions from merely apparent assertions, as in the mouth of an actor on the stage. For Wittgenstein there are *only* assertions, so there is no need of context to mark them off, or of any special sign to do so in the absence of context. Wittgenstein’s *Tractatus* would banish poets and actors as decisively as

Plato's *Republic* would. Incidentally, it should be remarked how singularly unsuccessful Frege's assertion sign would be in doing what it is supposed to do. An actor on the stage might write a message on a wall or a blackboard, and were Frege's views to become more widely known, the actor might even prefix his written message with the assertion sign—despite Professor Anscombe's prescribing such behaviour as "an inexcusable *faux pas*"! (I, 113) Or a person speaking only to check on whether some sound amplification equipment was working could just as easily prefix his non-asserted "Testing: one, two, three, four" with the words, "It is asserted that"—without thereby transmogrifying his utterance into an assertion.

Frege was sufficiently impressed by the difference between the non-assertive utterance of a component of a compound sentence, and the assertive utterance of the compound sentence containing it, to want to mark the difference by a special assertion sign. Wittgenstein might have replied that it was enough that each complete sentence was asserted and that no component—*qua* component—was asserted, with no need for any special sign to distinguish whole from part. But Wittgenstein went further than this, and in a highly original direction.

Wittgenstein denied that there are any components in the fully analysed forms of what we are accustomed to call "compound statements". The reasoning here turns on what Wittgenstein perceived to be an important distinction between two kinds of functions, that is, of what Frege had called "functions". These can be distinguished by differences in the relations of their domains and ranges. "Functions" whose values are included among their arguments, such as numerical valued functions of numbers, or truth functions of propositions, are called *operations* by Wittgenstein. "A truth-operation is the way in which a truth-function is produced out of elementary propositions" (IX, 5.3). Other functions, whose *values* are categorically different from their arguments, such as concepts and relations, are allowed to be proper functions. So mathematical operations and truth-functional connectives and operators are operations rather than functions. Wittgenstein sternly warns, "Operations and functions must not be confused with each other." (IX, 5.25)

The result of completing a function expression by inserting argument expressions in its blanks does contain those argument expressions. But the result of operating on a base need not contain that base, and the expression of the result need not contain an expression for that base, as the result of doubling the number 2 can be expressed simply as "4". Similarly, the result of negating the proposition " $\sim p$ " can be expressed simply as " p ". Thus "An operation can vanish (e.g. negation in ' $\sim\sim p$ ': $\sim\sim p = p$)." (IX, 5.254)

Now if the "compound sentence", that is, the result of operating truth functionally on one or more sentences, does not really *contain* any component sentences, then there is no longer any need for a special assertion sign to distinguish between an unasserted "B" in "If A then B" and an asserted "B". The conditional "If A then B" no longer need be regarded as literally *containing* "B" as component.

One can view the difference between Frege's and Wittgenstein's conceptions of logic and language in the following way. Scientific progress seems to be made for the most part by abstracting from the full richness and complexity found in "the state of nature". Galileo did not time apples rolling down hillsides. Instead he timed spheres made as nearly spherical as his technology permitted rolling down inclined planes machined and polished to as flat and smooth a surface as could be achieved. Thus he was able to formulate laws of falling bodies. But to apply those laws to apples rolling down hillsides is an enormously difficult task. Analogously, if logicians are to develop principles and methods for appraising the correctness or incorrectness of arguments, the arguments must be self-contained, isolated from the context of situation and presupposed background information needed to determine the significance of utterances in natural language. But Frege, for all his warnings about the way in which natural language is not "logical", (IV, 126) had the intention of analysing it according to the insights of his *Begriffsschrift*, and had the hope of directly applying *Begriffsschrift* results to it. For this to be accomplished it would indeed be necessary to be able to identify those utterances as assertions which can serve as ingredients of arguments. And for this purpose an assertion sign might be a useful device,

But in the *Tractatus* Wittgenstein was much more hard headed than Frege. In the development of logic Wittgenstein was willing to exclude all ethical and aesthetic considerations, and to restrict language to the language of natural science. There was no intention to apply logic to ordinary language, and, therefore, no need for any special assertion sign. From this point of view, since Frege and Wittgenstein had different intentions for the application of the symbolic logics they were developing, it is no wonder that they differed in their beliefs as to whether a special assertion sign was needed.

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NOTES

1. This paper was presented at the Golden Jubilee Session of the Indian Philosophical Congress at New Delhi, in Plenary Session IV on January 1, 1976. It was written while the author held a Fulbright-Hays Senior Research Fellowship in the United Kingdom in 1975.

2. References in the text are by capital Roman numerals to items in the list of References and by lower case arabic or roman numerals to the pages therein, except in the case of the *Tractatus* where the arabic numerals refer to the numbered passages of that work.

3. This translation of *Sachverhalt* is persuasively argued by Professor Stenius in his important and influential work on the *Tractatus* (VIII, 33).