Syllabus of
M.A./M. Sc. (Mathematics) Part II
University Courses

Semester III **Semester IV**

Compulsory Courses

MT-701. **Functional Analysis** MT-801. **Field Theory**

MT-702. **Ring Theory** MT-802. **Combinatorics**

Optional Courses (Any one out of three)

MT-703. **Mechanics** MT-803. **Differential Manifolds**

MT-704. **Measure and Integration** MT-804. **Algebraic Topology**

MT-705. **Graph Theory** MT-805. **Lattice Theory**

Departmental Courses (Any two)

MT 706 **Topics in Analysis-I**
MT 707 **Topics in Topology-I**
MT 708 **Topics in Operator Theory-I**
MT 709 **Operations Research-I**
MT 710 **Topics in Mechanics-I**
MT 711 **Topics in Relativity-I**
MT 712 **Complex Analysis-II**
MT 713 **Representation Theory of Groups**
MT 714 **Cryptography**
MT 715 **Differential Geometry**
MT 716 **Projective Geometry**
MT 717 **Algebraic Number Theory**
MT 718 **Topics in Matroid Theory**
MT 719 **Banach Algebra**
MT 720 **Boundary Value Problems**

MT 806 **Topics in Analysis-II**
MT 807 **Topics in Topology-II**
MT 808 **Topics in Operator Theory-II**
MT 809 **Operational Research-II**
MT 810 **Topics in Mechanics-II**
MT 811 **Topics in Relativity-II**
MT 812 **Fourier Analysis on Finite Groups**
MT 813 **Advanced Topics in Group Theory**
MT 814 **Coding Theory**
MT 815 **Computational Geometry**
MT 816 **Algebraic Geometry**
MT 817 **Commutative Algebra**
MT 818 **Advanced Topics in Matroid Theory**
MT 819 **Advanced Topics in Ring Theory**
MT 820 **Computational Mathematics**

Paper MT-701

FUNCTIONAL ANALYSIS

1. Banach Spaces

The definition and some examples.
Continuous linear transformations.
The Hahn-Banach theorem.
The natural imbedding of N in N**.
The open mapping theorem.
The conjugate of an operator.

2. Hilbert Spaces

The definition and some simple properties.
Orthogonal complements.
Orthonormal sets.
The conjugate space H*
The adjoint of an operator.
Self-adjoint operators.
Normal and unitary operators.
Projections.

3. Finite-Dimensional spectral Theory

Matrices.
Determinants and the spectrum of an operator.
The spectral theorem.
A survey of the situation.

Text Book :

G. F. Simmons , Introduction to Topology and Modern Analysis, McGraw Hill.
Chapters : 9, 10, 11.

Reference Books :

1. B. V. Limaye, Functional Analysis, Wiley Eastern Ltd.
2. Bachman and Narici, Functional Analysis.
3. W. Rudin, Functional Analysis, Tata Mc Graw Hill Edition.

Paper MT -702

RING THEORY

1. Preliminaries: Rings- Definition, Examples, Ring Homomorphism, Ideals, Ring of Fractions.

2. Euclidean Domains, P.I.D.'s, U.F.D.'s.

3. Polynomial Rings: Definition, properties, Polynomial Rings over Fields, Polynomial Rings that are U.F.D.'s, Irreducibility Criteria.

4. Noetherian Rings and Affine Algebraic sets, Radicals.

5. Artinian Rings, Discrete Valuation Rings and Dedikind Domains.

Text Book:
Dummit and Foote, Abstract Algebra, second edition (Wiley India).

Sections: 7.1 - 7.5, 8.1 - 8.3, 9.1 - 9.5, 10.1 -10.5, 15.1 –15.2, 16.1 –16.3 .

Reference Books:

1. C. Musili, Rings and Modules, 2nd Revised Edition, Narosa Publishing House.

2. Luther and Passi, Algebra II, Narosa Publishing House.

3. Jain and Bhattacharya, Basic Abstract Algebra, Second Edition, Cambridge University Press.

Paper MT -703

MECHANICS

Elementary principles: Mechanics of a particle, Mechanics of a system of particles, Constraints, D'Alembert's principle and Lagrange's equations. Simple applications of the Lagrangian formulation.

Variational principles and Lagrange's equations : Hamilton's principle, Some techniques of the calculus of variations, Derivation of Lagrange's equations from Hamilton's principle, Conservation theorem and symmetry properties.

The two-body central force problem : Reduction to the one-body equivalent problem, Equations of motion and first integrals. The virial theorem, The differential equation for the orbit, and integrable power-law potentials, The Kepler problem: Inverse square law of force, The motion in time in the Kepler problem.

The kinematics of rigid body motion : The independent co-ordinates of a rigit body, Orthogonal transformations, The Euler angles, Euler's theorem on the motion of a rigid body, Finite rotations, Infinitesimal rotations.

The Hamilton equations of motion : Legendre transformations and the Hamilton equations of motion, Cyclic co-ordinates and conservation theorems. Derivation of Hamilton's equations from a variational principle.

Canonical transformations : The equations of canonical transformation, Examples of canonical transformations, Poisson brackets.

Text Book :

Goldsten H., Classical Mechanics, Addition-Wesley, Second Edition, Narosa Publishing House, 2002.

Chapter 1 : 1-1, 1-2, 1-3, 1-4, 1-6, **Chapter 2 :** 2-1, 2-2, 2-3, 2-4, 2-6.
Chapter 3 : 3-1, 3-2, 3-4, 3-5, 3-7, 3-8. **Chapter 4:** 4-1, 4-2, 4-4, 4-6, 4-7, 4-8.
Chapter 8 : 8-1, 8-2, 8-5, **Chapter 9:** 9-1, 9-2, 9-4.

Reference Books :

1. Tiwari, R.N. and Thakur, B.S., Classical Mechanics, Prentice-Hall of India, New Delhi, 2007.

2. Gregory, R. Dougals, Classical Mechanics, Cambridge University Press, 2006.

3. Pars L. A., A treatise on analytical dynamics, London : Heinemann, 1965.

Paper MT -704

MEASURE AND INTEGRATION

Review : Lebesgue Measure and Lebesgue Integration.

1. Measure spaces, Measurable functions, Integration, General convergence theorems, Signed measure, The Radon-Nikodym Theorem, The L^P- spaces.

2. Outer measure and measurability, The extension theorem, The Lebgesuge-Stieltjes integral, Product measure, Inner measure, Extension by sets of measure zero, Caratheodory outer measure, Hausdorff measure.

3. Measure and Topology :
   Baire sets and Borel sets, The regularity of Barie and Borel measures, The construction of Borel measures.

Text Book :

H. L. Royden, Real Analysis (Pearson Education).

Chapters : 11, 12 (except 12.5), 13.1, 13.2, 13.3.

Reference Books :

1. P.R. Halmos , Measure Theory, Reprint (Springer-Verlag, 1974).

2. W. Rudin , Real and Complex Analysis, 3rd Edition, (Mc-Graw Hill).

3. C. D. Aliprants, O. Burkinshaw, Principles of Real Analysis ,(Harcourt Asia Pvt. Ltd.).

4. G. de Barra, Measure Theory and Integration, Wiley Eastern Ltd , 1981.

Paper MT -705

GRAPH THEORY

1. Fundamental Concepts :

Graphs, Matrices and isomorphism decomposition, connection in Graphs, bipartite graphs, Eulerian circuits, vertex degrees, and Graphic sequences.

2. Trees and Distance :

Trees, Distance in trees and Graphs, Enumeration of trees Caycley's formula, Spanning trees in graphs, minimum spanning trees, Kruskal's algorithm, shortest paths, Dijkstra's Algorithm.

3. Matchings :

Maximum Matchings, Hall's matching condition, Min-Max Theorems, Maximum bipartite Matching, weighted bipartite matching.

4. Connectivity and Paths :

Connectivity, edge-connectivity, blocks, 2-connected graphs, k-connected and k-edge-connected graphs, Menger's Theorem, Maximum Network flow, Max-flow min-cut Theorem.

Text Book :

West D.B. Introduction to Graph Theory (Second edition), Prentice Hall of India, New Delhi (2009).

Chapters : 1, 2, 3.1, 3.2, 4.

Reference Books :

1. J. Clark, D.A. Holton, A First Look at Graph Theory, Allied Publishers.

2. R. J. Wilson, Introduction to Graph Theory, (Fourth Edition), Pearson Education, Singapore (2003).

Paper MT -801

FIELD THEORY

1. Algebraic extensions of fields :

Irreducible polynomials and Eisenstein criterion.
Adjunction of roots.
Algebraic extensions.
Algebraically closed fields, Existence and Uniqueness of algebraic closure (without proof).

2. Normal and separable extensions :

Splitting fields.
Normal extensions.
Multiple roots.
Finite fields.
Separable extensions.

3. Galois Theory :

Automorphism groups and fixed fields.
Fundamental theorem of Galois theory.
Fundamental theorem of algebra.

4. Applications of Galois Theory to classical problems.

Polynomials solvable by radicals.
Ruler and compass constructions.

Text Book : P. Bhattacharya and S. Jain, Basic Abstract Algebra, Second Edition, Cambridge University Press.
Chapters : 15, 16, 17, 18.3, 18.5.

Reference Books :

1. John M. Howie, Fields and Galois Theory, Springer Undergraduate Mathematics Series.
2. Dummit and Foote, Abstract Algebra, 2nd Edition, Wiley Eastern Ltd.
3. M. Nagata, Theory of Field, Marcel Dekker.
4. O. Zariski and P. Sammuel, Commutative Algebra, Vol. 1, Van Nostrand.

Paper MT -802

COMBINATORICS

1. General Counting Methods :

Counting Principles, Arrangements and selections, Arrangements and selections with Repetitions, Distributions, Binomial Identities, Generating permutations and combinations.

2. Generating Functions :

Generating Function Models, Calculating coefficient of generating functions, Partitions, Exponential Generating Functions, A Summation Method.

3. Recurrence Relations:

Recurrence Relation Models, Divide and conquer Relations, Solution of Linear Recurrence Relations, Solution of Inhomogeneous Recurrence Relations, Solutions with Generating Functions.

4. Inclusion-Exclusion :

Counting with venn diagrams, Inclusion-Exclusion Formula, Restricted Positions and Rook polynomials.

5. Polya's Enumeration Formula :

Equivalence and Symmetry Groups, Burnside's Theorem. The Cycle Index, Polya's Formula.

Text Book :

Alan Tucker **:** Applied Combinations Fourth Edition (John Wiley and Sons, Inc).

Sections : 5.1 to 5.6, 6.1 to 6.5, 7.1 to 7.5, 8.1 to 8.3, 9.1 to 9.4, A4.

Reference Books :

1. V.K. Balkrishnan **:** Schaum's outline series. Theory and Problems of Combinations (Ms Graw Hill).
2. K.D. Joshi**:** Foundations of Discrete Mathematics (Wiley Eastern Limited).
3. Marshal Hall Jr.**:** Combinatorial Theory, Second Edition (Wiley Inter science Publications).

Paper MT -803

DIFFERENTIAL MANIFOLDS

Chapter 1 : Differential Manifolds

1. The volume of a Parallelopiped.
2. The volume of a Parametrized – Manifold.
3. Manifolds in $\mathbf{R}^n$.
4. The Boundary of a Manifold.
5. Integrating a Scalar Function over a Manifold.

Chapter 2 : Differential Forms

1. Multilinear Algebra.
2. Alternating Tensors.
3. The Wedge Product.
4. Tangent Vectors and Differential Forms.
5. The Differential Operator.
6. The Action of a Differentiable Map.

Chapter 3 : Stoke's Theorem

1. Integrating Forms over Parametrized-Manifolds.
2. Orientable Manifolds.
3. Integrating Forms over Oriented Manifolds.
4. The Generalized Stoke's Theorem.

Text Book :

James R. Munkres, Analysis on Manifolds, (Addision-Wesley Publishing Company).
Chapters : 5, 6, 7.

Paper MT -804

ALGEBRAIC TOPOLOGY

Homotopy; Homotopy type and Retractions, Paths, Path connectedness, The Fundamental group (Homotopy group), Fundamental group of the circle, Covering spaces; Fibrations, simplexes and complexes, Simplicial Homotopy Theory.

Text Book:

1. B. K. Lahiri , A First course in Algebraic Topology (Second Edition), Narosa Publishing House, (2005).

Chapters : 3-11.

References:

1. M.A. Armstrong, Basic Topology, Springer Verlag (2004)(Chapters 5 and 8).
2. Munkres J. R., Topology, Prentice Hall (1975).

Paper MT -805

LATTICE THEORY

1. **Lattices :**

   (a) Equivalence of two definitions.
   (b) Homomorphisms
   (c) Sublattices, ideals and congruence relation.
   (d) Product of lattices
   (e) Polynomial identities in lattices.
   (f) Distributive and Modular lattices
   (g) Special elements in lattices.

2. **Characterization and Representation :**
   (a) Dedekinds modularity ceiterion.
   (b) Bickhoff's distributivity criterion.
   (c) Stone representation theorem.
   (d) Machbin Theorem.
   (e) Hahimoto's Theorem.

3. **Modular and Semimodular lattices:**

   (a) Isomorphism Theorems.
   (b) Upper and lower covering conditions.
   (c) Semimodular lattices.
   (d) Jordan-Holder chain condition.

4. **Complete Lattice :**

   (a) Closure Operations
   (b) Embedding in complete lattices.
   (c) Conditional Completeness.
   (d) Fixpoint Theorem.

Text Books :

1. G. Gruatzer, General Lattice Theory, Academic Press, 1978.
   Chapters : 1, 2 (Section 1), 3 (section 1).

Reference Books :

1. G. Baskhoff , Lattice Theory, 3rd Edition, American Mathematical Society, 1940.
2. D. E. Rutherford, Introduction to Lattice Theory, Oliver and Boyd, London, 1965
3. G. Szasz, Introduction to Lattice Theory, Academic Press, New York, 1963.