

University of Pune  
M.A. and M.Sc.(Semester Pattern)- Part I

Notes

- (1) Each theory course prescribed for M.A./M.Sc (Statistics) – semester pattern should be covered in 4 periods, each of 60 minutes duration per week per course including lectures, tutorials, seminars etc.
- (2) Each practical course will require 6 hours of laboratory work per week and the course will be examined at the end of each semester.
- (3) There should not be more than 10 students in a batch for M.Sc. practical course.
- (4) Out of 60 lectures in each course about 10 lectures will include tutorials students seminars and class tests.
- (5) Two interactive sessions per course per semester must be conducted by concerned teachers.
- (6) Each theory course will be examined for out of 100 marks of which 80 marks are reserved for final examination at the end of the semester and 20 marks for internal assessment.
- (7) Final examination of 80 marks will be of 3 hours duration.
- (8) There shall be mid-semester internal tests of 15 marks, and viva and or seminar of 5 marks for each theory course.
- (9) Final practical examination will be of 3 hours duration. It will be for 80 marks of which 10 marks will be reserved for viva at the time of examinations.
- (10) There shall be 20 marks for internal evaluation for each practical course. 10 marks are reserved for journal and 10 marks for viva at the completion of every experiment.
- (11) For departmental course college/institute should appoint **one external paper setter or examiner** for the final examination.

## Titles of Papers

### SEMESTER - I

- 1) ST - 11 Mathematical Analysis
- 2) ST - 12 Linear Algebra
- 3) ST – 13 Probability distribution
- 4) ST - 14 Sampling Theory and Computational Techniques
- 5) ST – 15 Practical (Departmental course)

### SEMESTER – II

- 1) ST- 21 Probability theory
- 2) ST -22 Regression Analysis
- 3) ST -23 Statistical Inference I
- 4) ST- 24 Elementary Stochastic Processes
- 5) ST -25 Practical (Departmental course)

## ST-11 : MATHEMATICAL ANALYSIS

- (i) Introduction to  $n$  dimensional Euclidean space and metric space, system of real numbers, countable and uncountable sets, countability of rational numbers, uncountability of real numbers, supremum and infimum of set of real numbers, interior point, limit point of a set, open set, closed set, dense and compact sets, Bolzano – Weierstrass theorem, Heine – Borel theorem (Statement only), applications of these theorems. [10]
- (ii) Sequence of real numbers, limit of a sequence and its properties, convergence and divergence, Cauchy sequence, subsequence, convergence of bounded monotone sequence, limit inferior, limit superior of a sequence, properties of limsup and liminf [8]
- (iii) Series of real numbers, convergence and divergence, test for convergence (without proof), absolute convergence, conditional convergence, test for absolute convergence (without proof), uniform convergence, convergence of series of nonnegative terms, applications to discrete distributions [6]
- (iv) Real valued function, continuity, uniform continuity, power series, uniform convergence, radius of convergence of binomial, exponential, geometric, log and trigonometric series, term by term differentiation and integration, functions of several variables, differentiability, mean value theorem, Taylor's theorem, generalization to several variables (Statements only), applications of these in mgf, maxima and minima, constrained maxima and minima [10]
- (v) Riemann and Riemann –Stieltjes integral, application in statistics, Cauchy-Schwartz inequality, integration by parts, Differentiation under integral sign (without proof) multiple integral, definition, Fubini's theorem on interchange multiple integral, definition, Fubini's theorem on interchange of order of integration [8]
- (vi) Improper integrals of type I and type II, conditions for convergence of beta and gamma functions, relation between beta and gamma functions, properties of beta and gamma functions, duplication formula [6]

**Books :**

- (1) Apostol, T. M. (1975), Mathematical Analysis: A modern approach to advanced calculus (Addison-Wesley)
- (2) Bartle, R. G. (1976), Elements of real analysis (Wiley)
- (3) Goldberg R.R. (1964), Methods of Real Analysis (Bailssdell Publishing Company)
- (4) Malick S. C. and Arora S. (1991), Mathematical Analysis (Wiley Eastern, IIInd edn)
- (5) Rudin, W. (1985), Principles of mathematical analysis (McGraw Hill)
- (6) Kreyszig, E. (1975), Advanced Engineering Mathematics (Wiley Eastern)

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**ST- 12 LINEAR ALGEBRA**

- (i) Vector space, subspace, linear dependence and independence, basis of vector

- space, dimension of a vector space, examples (4)
- (ii) Matrix algebra, special types of matrices , elementary operations, rank of a matrix, orthonormal basis and orthogonal projection of a vector, Gram-Schmidt orthogonalization, projection theorem, Kronecker product, idempotent matrix, inverse of a matrix, its simple properties, partitioned matrices (12)
- (iii) Characteristic roots of a matrix, right and left characteristic vectors, Properties of characteristic roots and vectors, algebraic and geometric multiplicities, spectral decomposition,  $n^{\text{th}}$  power of a matrix, Cayley-Hamilton theorem (12)
- (iv) G-inverse, Moore-Penrose g-inverse, solution of a system of homogeneous and non-homogeneous linear equations (10)
- (v) Quadratic forms : definition, reduction and classification, simultaneous reduction of two quadratic forms, maxima and minima of ratio of quadratic forms. (10)

### **Books:**

- (1) Searle S.A.(1982) Matrix Algebra Useful for Statistics (Wiley)
- (2) Graybill (1961) An introduction to linear Statistical models Vol.I (Mc Graw Hill )
- (3) Rao C.R. (1973) Linear Statistical Inference and its Applications (Wiley Eastern)
- (4) Hadley G. (1987) Linear Algebra (Narosa)
- (5) Rao, C.R. and Bhimashankaram, P. (1992) Linear Algebra (Tata Mc Graw Hill)

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## **ST 13 : PROBABILITY DISTRIBUTIONS**

- (i) Brief review of a random variable, c.d.f and its characteristic properties

with proof for univariate and bivariate probability distributions, quantiles, discrete, continuous distributions, p.m.f., p.d.f., symmetric distributions, mixtures of probability distributions, transformation of random variables, m.g.f, moments, p.g.f (8)

- (ii) Random vectors, joint probability distributions, joint m.g.f., mixed moments, variance covariance matrix, independence, sums of independent r.v.s, convolutions, conditional expectation and variance, regression function and best linear regression function (8)
- (iii) Bivariate Poisson, bivariate exponential (all 4 types), bivariate normal distribution, Dirichlet distribution and their properties (8)
- (iv) Exponential class of distributions, location and scale families, non-regular families
- (v) Review of order statistics, distribution of rth order statistic, joint distribution of two order statistics and their functions, probability integral transformation, distribution of rank factors, distributions of sign statistic, Kolmogorov-Smirnov statistic and Wilcoxon sign rank statistic (16)
- (vi) Sampling distributions of statistics for random samples from normal distribution, Fisher Cochran theorem, Non-central chi-square, t, F distribution (8)

### **Books :**

- (1) Rohatagi V.K. & Saleh A.K.(2001) Introduction to probability theory and mathematical statistics. (John Wiley and sons)
- (2) Johnson N.L. & Kotz S.(1996) Distributions in statistics Vol.I .VolII and Vol III John Wiley and sons Inc.)
- (3) Johnson N.L., Kotz S., Balkrishnan, N. Multivariate Distributions (John Wiley and sons )
- (4) Casella and Burger(2002) Statistical Inference (Duxbury advanced series II edition)
- (5) Hogg RV and Craig T T (1978) Introduction to mathematical Statistics 4<sup>th</sup> edn. (Collier McMillan)
- (6) Rao, C R (1995) Linear statistical inference and its applications 2<sup>nd</sup> edn (Wiley Eastern)
- (7) Gibbons, J D and Chakraborty, S (1992) Nonparametric statistical inference 3<sup>rd</sup> edn.(Marcell Dekker)

(8) Conover W J (1999) Practical nonparametric statistics 3<sup>rd</sup> edn, (Wiley)

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## **Section I : Sampling theory**

- (i) Review of basic finite population sampling techniques (SRSWR, SRSWOR, stratified sampling, systematic sampling) and related results on estimation of population mean/ total, problem of allocation in stratified sampling, review of ratio and regression methods, construction of strata, deep stratification, method of collapsed strata (8)
- (ii) Probability proportional sampling(PPS) with replacement::cumulative total method, Lahiri's method for estimation problem, estimation of a finite population mean, PPSWOR: Hurwitz-Thompson estimator and its properties. (10)
- (iii) Cluster sampling, estimation of population mean and its standard error, multistage sampling: double sampling (2)
- (iv) Sampling and non-sampling errors, randomized response techniques (2)

## **SECTION II : COMPUTATIONAL TECHNIQUES**

- (i) Introduction to R, data structures in R, input and output in R, statistical functions in R, writing simple programs (6)
- (ii) Solutions to algebraic equations, bisection method, NR method, steepest descent, quadrature interpolation, Jacobi and Gauss Seidel methods (8)
- (iii) Unconstrained optimization methods, direct search, grid search, Hooke Jeeves' method, interpolatory search, gradient search (6)
- (iv) Random variable generation method, mixed congruential, multiplicative congruential, rejection, distribution specific methods (6)

### **Books :**

- (1) Sukhatme P.V., Sukhatme B.V. and C. Asok Sampling theory of survey and applications (Indian society for Agricultural statistics )



- (2) Des Raj & Chandhok P.(1998), Sample survey theory (Narosa)
- (3) W. G. Cochran ,(1977) Sampling techniques (John Wiley and sons)
- (4) Murthy M.N.(1977) Sampling theory and methods (Statistical Publishing Society)
- (5) Thisted R A (1988) Elements of Statistical Computing (Chapman Hall)  
(Rajaraman V. (1993) Computer oriented numerical methods 4<sup>th</sup> edn (Prentice Hall)
- (6) Ross S (2000) Introduction to probability model (Academic Press)

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### **ST 15 : PRACTICAL (Departmental Course)**

1. Determinant and rank of a matrix

2. Inverse of a square and nonsingular matrix, inverse by partitioning of a matrix
3. g-inverse, MP g-inverse
4. Eigen values, eigen vectors, spectral decomposition, power of a matrix
5. Solution of simultaneous equations
6. Classification and reduction of quadratic forms
7. Plotting of density functions and distribution functions
8. Model sampling from probability distributions
9. Model sampling from mixtures of distributions
10. Computation of probabilities of events related to bivariate probability distributions
11. Computation of probabilities of events related to bivariate probability Distributions II
12. Computation of probabilities of non-central chi-square, t, F distributions
13. PPS design
14. Double sampling
15. Cluster sampling, two stage sampling
16. Systematic sampling
17. Randomized response techniques
18. Calculation of summary statistic using R
19. Calculation of double integral
20. Computation of integrals by Riemann and RS sums.

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## **ST 21 : PROBABILITY THEORY**

- (i) Review of algebra of sets, sequence of sets , limsup, liminf and limit of a sequence of sets, field, sigma field, minimal sigma field, Borel fields, measurable space (6)

- (ii) Probability measure on a measurable space, probability space, properties of probability measure: continuity, mixture of probability measures, Lebesgue and Lebesgue-Stieltjes measures (4)
- (iii) Measurable function, Real and vector valued random variables, simple r.v., r.v. as a limit of sequence of simple r.v.s, discrete and continuous type r.v., distribution function, decomposition of a distribution function (8)
- (iv) Integration of a measurable function wrt a measure, expectation of a r.v., linear properties of expectation, characteristic function and its simple properties, Parseval relation, uniqueness theorem, inequalities of moments (6)
- (v) Convergence of a sequence of r.v.s, almost sure convergence, convergence in quadratic mean, convergence in probability, convergence in distribution, their inter-relations
- (vi) Independence of two and  $n (>2)$  events, sequence of independent events,  $\pi$  and  $\lambda$  system of events, Dynkin's theorem, independence of r.v.s, Kolmogorov zero-one law, Borel Cantelli lemma (6)
- (vii) Monotone convergence theorem, dominated convergence theorem, Levy continuity theorem(statement only), Khintchin's WLLN, Kolmogorov's SLLN(without proof) (6)
- (viii) CLT for i.i.d. r.v.s, Liapounev's form, Lindeberg Feller form and their applications (6)

**Books :**

- (1) Bhat B.R.(1985) Modern Probability theory (Wiley Eastern )
- (2) Billingsley P. (1986) Probability and Measure (Wiley)
- (3) Feller W. (1969) Introduction to probability and its applications Vol.II (Wiley Eastern)

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**ST-2 2 : REGRESSION ANALYSIS**

- (i) Simple linear regression, assumptions, least square (LS) estimators of parameters, s.e. of estimators, testing of hypothesis for coefficient of regression, s.e. of prediction, testing of hypotheses about parallelism, equality of intercepts, congruence,

extrapolation, optimal choice of  $X$ , , diagnostic checks and correction: graphical technique, tests for normality , uncorrelated ness, homoscedasticity, lack of fit. modifications like polynomial regression, transformations on  $Y$  or  $X$ , weighted LS, inverse regression.

(8)

- (ii) Multiple regression: Standard Gauss-Markov setup, least square estimation, error and estimation spaces, variance and covariance of LS estimators, properties of LS estimators, estimation of error variance, case with correlated observation, LS estimation with restriction on parameters, simultaneous estimation of linear parametric functions, testing of hypothesis for one and more than one linear parametric functions, confidence intervals and regions (8)
- (iii) Multiple correlation, adjusted multiple correlation coefficient, null distribution of simple correlation and multiple correlation coefficient, partial correlation coefficient and its relation with multiple correlation coefficient, test for significance of simple , multiple and partial correlation coefficients, variable selection procedures, Mallows  $C_p$ , forward, backward selection method (10)
- (iv) Residual and residual diagnostics, transformation of variables: Box-Cox power transformation (6)
- (v) Multicollinearity : consequences, detection and remedies, autocorrelation : consequences, Durbin Watson test, estimation of parameters in autocorrelation (6)
- (vi) Non-linear regression: Non-linear least squares transformation to a linear model, statistical inference in non-linear regression (4)
- (vii) Generalized linear model : Logistic regression: Logit transform, ML estimation, tests of hypothesis, Wald test, LR test, score test, test for overall regression, introduction to link functions such as binomial, inverse binomial, inverse Gaussian and Gamma. (6)

### **Books:**

- (1) Draper, N. R. and Smith H. (1998) Applied regression analysis 3<sup>rd</sup> edition (John Wiley)

- (2) McCullagh, P. and Nelder, J. A.(1989) Generalized linear models (Chapman and Hall)
- (3) Ratkowsky, D. A.(1983) Nonlinear regression modeling (Marcel Dekker)
- (4) Hosmer, D. W. and Lemeshow, S. (1989) Applied logistic regression(John Wiley)
- (5) Neter, J.; Wasserman, W. and Kutner, M.H.(1985) Applied linear statistical models
- (6) Montgomery D.C. et. al.(2003) Introduction to linear regression analysis (Wiley Eastern)

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## **ST-23 STATISTICAL INFERENCE I**

- (i) Sufficiency :factorization theorem, minimal sufficiency, minimal sufficient partition, construction of minimal sufficient statistics, special classes of distributions admitting complete sufficient statistic (8)
- (ii) Completeness, bounded completeness, special classes of distributions admitting complete sufficient statistic ancillary statistic, Basu's theorem and its applications, unbiased estimator, UMVUE, Rao-Blackwell theorem, Lehman-Scheffe theorem and their uses, Fisher information and information matrix, CR inequality, regularity conditions, Chapman-Robins bound, Bhattacharya bound(statement only) (12)
- (iii) Test function, NP lemma (with proof) for test function, UMP test for one-sided alternative for one parameter exponential class of densities and extension to the distribution having monotone likelihood ratio property , statement of UMPU test one parameter exponential family (10)
- (iv) Confidence interval (C.I.), relation with testing of hypothesis, uniformly most accurate C.I., shortest expected length C.I. (6)
- (v) Introduction to Bayesian estimation: prior and posterior distribution, loss Function, principle of minimum expected posterior loss, quadratic and other common loss functions, conjugate family of prior distributions and its examples. (12)

### **Books:**

- (1) Kale B.K(1999) A First course on Parametric Inference ( Narosa )
- (2) Casella G. & Beregar R.L.(2002) Statistical Inference, 2<sup>nd</sup> edition, (Duxbury Advanced series)
- (3) Dudewitz E.J. & Mishra S.N.(1988) Modern mathematical Statistics (John Wiley)
- (4) Lehman E.L.(1988) Theory of point estimation (John Wiley)
- (5) Lehman E.L.(1986) Testing of statistical hypotheses (John Wiley)
- (6) Rohatagi V.K.(1976) Introduction to theory of probability and mathematical statistics (John Wiley and sons)

## **ST 24 : STOCHASTIC PROCESSES**

- (i) Markov chain with stationary transition probabilities, Chapman Kolmogorov equation, n-step transition probabilities, classification of states (persistent, transient, nonnull, null persistent, periodic, aperiodic), ergodic Markov chain, stationary distribution of Markov chain, existence and uniqueness of stationary distribution, interpretation of stationary probability (8)
- (ii) Random walk, random walk with absorbing and reflecting barrier, classification of states, probability of absorption in persistent class starting from transient state, application to gambler's ruin problem: probability of ruin, expected gain, expected duration of a game (6)
- (iii) BGW branching process, mean and variance, generating function for probability of ultimate extinction, nth generation size and related recurrence relations (6)
- (iv) Markov pure jump process, continuous time Markov process, Kolmogorov forward and backward equations, Poisson process, pure birth process, birth and death process, properties and results associated with these processes (8)
- (v) Introduction to renewal process, key renewal and elementary theorem (without proof), renewal equations and its solution (without proof), Poisson process as a renewal process, applications of renewal process in reliability theory (8)
- (vi) Elementary queuing models: M/M/1, M/M/K, M/G/K queuing systems, relationship between queuing models and birth and death process, applications of these to real life problems (8)
- (vii) Introduction to Brownian motion, Weiner process and its properties (4)



**Books:**

- (1) Medhi, J. (1982) Stochastic processes (Wiley Eastern)
- (2) Taylor, H N and Karlin, S. (1984) An introduction to stochastic modeling (Academic Press)
- (3) Ross, S. (2000) Introduction to probability models, 7<sup>th</sup> edn (Academic Press)
- (4) Srinivas and Mehta (1976) Stochastic Processes (Tata Mc-Graw Hill)
- (5) Ross, S. : Stochastic processes (John Wiley)
- (6) Adke, S.R. and Manjunath, S.M. (1984) An introduction to finite Markov processes (Wiley Eastern)
- (7) Bhat, B.R. (2000) Stochastic models: Analysis and applications (New Age International)
- (8) Athreya and Lahiri : Probability theory

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## **ST 25: PRACTICAL (Departmental Course)**

- 1 Simple regression and regression diagnostic I
- 2 Simple regression and regression diagnostic II
- 3 Multiple regression forward and backward method I
- 4 Multiple regression forward and backward method II
- 5 Nonlinear regression I
- 6 Nonlinear regression II
- 7 Logistic regression I
- 8 GLM and linearization transforms
- 9 MP test
- 10 UMP test
- 11 Applications of central limit theorem
- 12 Confidence intervals
- 13 Bayesian methods I
- 14 Bayesian methods II
- 15 Realization of Markov chain when t.p.m. is given
- 16 Computation of transition probabilities of Markov chain based on realization
- 17 Verification of Chapman Kolmogorov equations
- 18 Simulation of branching process and computation of extinction probability
- 19 Stationary distribution of a Markov chain
- 20 Realization of Poisson process and queuing models

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