University of Pune
Board of Studies in Mathematics
Syllabus for T. Y. B. A.(Mathematics)

AMG-3	Real Analysis and Lebesgue Integration
MG-3	Group Theory and Ring Theory
MS-3	Set Theory, Logic and Metric Spaces
MS-4	Ordinary and Partial Differential equations
FMG-3	C-programming

AMG-3: Real Analysis and Lebesgue Integration
First Term: Real Analysis

1. **S**equences of real numbers : Definition of sequence and subsequence, Limit of a sequence, convergent sequences, Limit superior and Limit inferior, Cauchy sequences. **[10 Lectures]**

2. **S**eries of Real numbers : Convergence and divergence of series of real numbers, alternating series, Conditional and absolute convergence of series, test of absolute convergence (Ratio test and Root test), series whose terms form a non-increasing sequence. **[10 Lectures]**

3. **R**iemann integral : Sets of measure zero, Definition and existence of a Riemann integral, properties of Riemann integral, Fundamental theorem of integral calculus, Mean value theorems of integral calculus. **[14 Lectures]**

4. **S**equence and series of functions : Pointwise and uniform convergence, sequence of functions, consequences of uniform convergence, convergence and uniform convergence of series of functions, integration and differentiation of series of functions. **[14 Lectures]**

Text Books:

1. R.R. Goldberg - Methods of Real Analysis (Oxford and IBH Publications (1970)).
   Ch. 2 Art. 2.1, 2.9, 2.10.
   Ch. 3 to 3.3, 3.4A, 3.4B, 3.6F, 3.6G, 3.7.
   Ch. 7 Art. 7.1 to 7.4, 7.8 to 7.10.
   Ch. 9 Art. 9.1 to 9.5

Reference Books:

1. D. Somasundaram, B. Choudhary - A first course in Mathematical Analysis, Narosa Publishing House, 1997.

2. Robert, G. Bartle, Donald Sherbert - Introduction to real analysis, Third edition, John Wiley and Sons.

3. Shantinarayan and Mittal - A course of Mathematical Analysis, Revised edition, S. Chand and Co.(2002).

4. S.C. Malik and Savita Arora - Mathematical Analysis , New Age International Publications,Third Edition,(2008).

Second term: Lebesgue Integration

1. **Measurable Sets** **[12 Lectures]**
   (i) Length of open sets and closed sets.
   (ii) Inner and outer measure.
   (iii) Measurable sets.
   (iv) Properties of measurable sets.

2. **Measurable Functions** **[12 Lectures]**

3. **The Lebesgue integrals** **[16 Lectures]**
   (i) Definition and example of the Lebesgue integrals for bounded functions.
   (ii) Properties of Lebesgue integrals for bounded measurable functions.
   (iii) The Lebesgue integral for unbounded functions.
   (iv) Some fundamental theorems.

4. **Fourier Series** **[8 Lectures]**
   (i) Definition and examples of Fourier Series.
   (ii) Formulation of convergence problems.

Text-Book:
Richard R. Goldberg, Methods of Real Analysis, Oxford and IBH Publishing Co. Pvt. Ltd. (1970).
(Chapter No. 11, 11.1 to 11.8, 12.1, 12.2. Theorem No. 11.1B and 11.1C, 11.8D Statements only).

Reference Books:

1. Tom Apostol, Advanced Calculus, 2nd Edition, Prentice Hall of India, (1994).

2. D. Somasundaram and B. Choudhari, A first course in Mathematical Analysis, Narosa Publishing House, (1997).

3. R.G. Bartle and D.R. Scherbert, Introduction to real analysis 2nd Edition, John Wiley, (1992).

4. Inder K. Rana, Measure and Integration

MG-3: Group Theory and Ring Theory
First term: Group Theory

Groups **[12 Lectures]**

1. Groups : definition and examples.
2. Abelian group, finite group, infinite group.
3. Properties of groups.
4. Order of an element - definition, examples, properties.
5. Examples of groups including $\mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$, Klein 4-group, Group of quaternions, S^1(= the unit circle in $\mathbb{C}$), $GL_n(\mathbb{R}), SL_n(\mathbb{R}), O_n$(=the group of $n \times n$ real orthogonal matrices), B_n(= the group of $n \times n$ nonsingular upper triangular matrices), and groups of symmetries of plane figures such as D_4 and S_3.

Subgroups **[10 Lectures]**

1. Subgroups : definition, necessary and sufficient conditions, examples on finding subgroups of finite groups, union and intersection of subgroups.
2. Subgroup generated by a subset of the group.
3. Cyclic groups : definition, examples of cyclic groups such as $\mathbb{Z}$ and the group μ_n of the n-th roots of unity, properties :
   (a) Every cyclic group is abelian.
   (b) If $G = (a)$, then $G = (a^{-1})$.
   (c) Every subgroup of a cyclic group is cyclic.
   (d) Let G be a cyclic group of order n. Let $G = (a)$. The element $a^s \in G$ generates a cyclic group of order $\dfrac{n}{\gcd(n, s)}$.
   (e) Let $G = (a)$ and $o(G) = n$. Then $(a^m) = G$ if and only if $(m, n) = 1$.
4. Cosets : definition and properties.
5. Lagrange's theorem and corollaries.

Permutation Groups **[6 Lectures]**

1. Definition of S_n and detail discussion of the group S_3.
2. Cycles and transpositions, even and odd permutations.
3. Order of permutation.

4. Properties : (i) $o(S_n) = n!$ (ii) A_n is a subgroup of S_n.

5. Discussion of the group A_4 including converse of Lagrange's theorem does not hold in A_4.

Normal Subgroups **[8 Lectures]**

1. Definition.

2. Properties with examples:

   (a) If G is an abelian group, then every subgroup of G is a normal subgroup.

   (b) N is a normal subgroup of G if and only if $gNg^{-1} = N$ for every $g \in G$.

   (c) The subgroup N of G is a normal subgroup of G if and only if every left coset of M in G is a right coset of N in G.

   (d) A subgroup N of G is a normal subgroup of G if and only if the product of two right cosets of N in G is again a right coset of N in G,.

   (e) If H is a subgroup of index 2 in G then H is a normal subgroup of G.

   (f) If H is the only subgroup of G of a fixed finite order then H is a normal subgroup of G.

3. Quotient groups and examples.

Homomorphism and Isomorphism **[12 Lectures]**

1. Homomorphism.

2. Isomorphism : definition, examples, establish isomorphism of two finite groups.

3. Fundamental Theorem of homomorphisms of groups.

4. The group $\mathbb{Z}/n\mathbb{Z}$ of residue classes (mod n). Characterization of cyclic groups (as being isomorphic to $\mathbb{Z}$ or $\mathbb{Z}/n\mathbb{Z}$ for some $n \in \mathbb{N}$).

5. Cayley's Theorem for finite groups.

6. Classification of groups of order ≤ 5.

7. Cauchy's theorem for Abelian Groups.

Text book:
I.N. Herstein, Topics in Algebra, Wiley, 1990.

Reference Books :

1. M. Artin, Algebra, Prentice Hall of India, New Delhi, 1994.

2. P.B. Bhattacharya, S.K. Jain and S.R. Nagpal, Basic Abstract Algebra, Second Ed., Foundation Books, New Delhi, 1995.

3. J.B. Fraleigh, A. First Course in Abstract Algebra, Third Ed., Narosa, New Delhi, 1990.

4. N.S. Gopalakrishnan, University Algebra, Second Ed., New Age International, New Delhi, 1986.

5. D.A.R. Wallace, Groups, Rings and Fields, Springer-Verlag, London, 1998.

6. I.N. Herstein, Abstract Algebra.

7. I. H. Sheth, Abstract Algebra, Second Revised Edition, 2009, PHL,India.

Second term: Ring Theory

1. Definition and properties of Ring, Subring. **[5 Lectures]**

2. **Integral Domains**: Zero divisors, Cancellation Law, Field, Characteristics of Ring. **[5 Lectures]**

3. **Ideals and Factor Rings**: Existence of Factor Ring, Prime Ideals, Maximal Ideals. : **[6 Lectures]**

4. **Homomorphism of Rings**: Properties of Ring Homomorphism, Kernel, First isomorphism Theorem for Ring, Prime Fields. The field of Quotients. **[8 Lectures]**

5. **Polynomial Ring**: Definition. The division Algorithm, Principle Ideal Domain. **[6 Lectures]**

6. **Factorization of Polynomial**: Reducibility and Irreducibility Tests, Eisenstein criterion. Ideals in $F[x]$. Unique Factorization in $Z[x]$. **[8 Lectures]**

7. **Divisibility in Integral Domain**: Associates, Irreducible and Primes, Unique Factorization Domains, Ascending chain Condition for PID, PID implies UFD, Euclidean Domains. ED Implies PID,
D is UFD implies $D[x]$ is UFD. **[10 Lectures]**

Text Book:
Joseph, A. Gallian, Contemporary Abstract Algebra,(4th Edition), Narosa Publishing House.
Chapter Numbers : 12,13,14,15,16,17 and 18.

Reference Books:

1. J.B. Fraleigh, First course in Abstract Algebra (4rd Edition). Narosa Publishing House.

2. I.N. Herstein. Abstract Algebra, (3rd Edition), Prentitice Hall of India, 1996.
3. N.S. Gopalkrishnan, University of Algebra, Wiley Eastern 1986.
4. C. Musili, Rings and Modules, Narosa Publishing House, 1992.

MS-3: Set Theory, Logic and Metric Spaces
First term: Set Theory and Logic

Sets and Relations : Cantor's concept of a set, Intuitive set theory, Inclusion, Operations for sets, Algebra of sets, Equivalence relations, Functions, Composition and Inversion of Functions, Operations for collections of sets, Ordering relations, Power sets, Numerical Equivalence of sets. **[8 Lectures]**

Natural Number sequence :
Induction and Recursion, Cardinal numbers and Cardinality, Cardinal arithmetic, Countable and Uncountable sets, Schroeder-Bernstein Theorem (without proof), Paradoxes of Intuitive set theory, Russell's Paradox. **[12 Lectures]**

Logic :
Statement calculus (Sentential connectivities, Truth tables, Validity, Consequence, Applications), Predicate Calculus (Symbolizing every day language, Formulation, Validity, Consequence). **[4 Lectures]**

Basic Logic :
(Revision) Introduction, proposition, truth table, negation, conjunction and disjunction, Implications, biconditional propositions, converse, contra positive and inverse propositions and precedence of logical operators. **[6 Lectures]**

Propositional equivalence :
Logical equivalences, Predicates and quantifiers : Introduction, Quantifiers, Binding variables and Negations. **[6 Lectures]**

Methods of Proof:
Rules of inference, valid arguments, methods of proving theorems; direct proof, proof by contradiction, proof by cases, proofs by equivalence, existence proofs, Uniqueness proofs and counter examples. **[12 Lectures]**

Text Books:

1. Set Theory and Logic, Robert R. Stoll, Errasia publishers, New Delhi. Sections 1.1 to 1.10, 2.3, 2.4, 2.5

2. Discrete Mathematics and its Applications, K.H. Rosen, Tata McGraw, New Delhi. Chapter 4

Reference Books :

1. Symbolic Logic, I.M. Copi, Fifth Edition, Prentice Hall of India, 1995.

2. Naive Set Theory, P.R. Halmos, 1974.

Second Term: Metric Spaces

1. Chapter 1 : Basic Notions. **[8 Lectures]**
2. Chapter 2: Convergence. **[8 Lectures]**
3. Chapter 3 : Continuity. **[8 Lectures]**
4. Chapter 4 : Compactness. **[10 Lectures]**
5. Chapter 5 : Connectedness. **[6 Lectures]**
6. Chapter 6 : Complete Metric Spaces. **[8 Lectures]**

Text Book:
Topology of Metric Spaces by S. Kumaresan, Narosa Publishing House, 2005.

Sections : 1.1, 1.2 (except the Sections 1.2.51 to 1.2.65), 2.1, 2.2, 2.3, 2.4, 2.5 and 2.7, 3.1, 3.2 (up to 3.2.32 only), 3.3, 3.4,3.5.(Uniform Continuity to be dropped), 4.1, 4.2, (Proposition 4.2.13 without proof) and 4.3 (Theorem 4.3.24 without proof), 5.1 and 6.1 (Theorems 6.1.1, 6.1.3, 6.1.11, without proofs).

Note: All the problems which are based on normed linear spaces and matrices be dropped.

Reference books :

1. Real Analysis, Carothers, Cambridge University Press, 2000.
2. Methods of Real Analysis, R.R. Goldberg, Oxford and IBH Publishing Company.
3. Metric Spaces, E.T. Copson, University Press, Cambridge, 2nd edition, Mumbai, 1978.
4. Introduction to Topology and Modern Analysis, G.F. Simmons. McGraw Hill International Book Company, International Student Edition.

MS-4: Ordinary and Partial Differential equations

First term: Ordinary Differential Equations

1. What is a Differential Equation?: **[14 Lectures]**
   Introductory Remarks, the nature of solutions, separable equations, first-order linear equations, exact equations, orthogonal trajectories and families of curves, homogeneous equations, integrating factors,
   reduction of order:(1) dependent variable missing, (2) independent variable missing, electrical circuits.

2. Second-Order Linear Equations: **[12 Lectures]**
   Second-order linear equations with constant coefficients, the method of undetermined coefficients, the method of variation of parameters, the use of a known solution to find another, vibrations and oscillations : (1) undamped simple harmonic motion (2) damped vibrations (3) forced vibrations.

3. Power Series Solutions and Special Functions: **[12 Lectures]**
   Introduction and review of power series, series solutions of first-order differential equations, second-order linear equations, ordinary points, regular singular points, more on regular singular points.

4. System of First-Order Equations: **[10 Lectures]**
   Introductory remarks, linear systems, homogeneous linear systems with constant coefficients.

Text Book: Differential Equations by George F. Simmons, Steven G. Krantz, Tata McGraw-Hill.

Reference Book:

1. W.R. Derrick and S.I. Grossman, A First Course in Differential Equations with Applications. CBS Publishers and distributors, Delhi-110 032. Third Edition.

2. Rainville, Bedient: Differential Equations

Second Term: Partial Differential Equations

1. **Ordinary Differential Equations in More Than Two Variables**

   (a) Surface and Curves in Three Dimensions **[20 Lectures]**

   (b) Simultaneous Differential Equations of the First Order and the First Degree in Three Variables.

   (c) Methods of solution of $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$.

   (d) Orhogonal Trajectories of a System of curves on a Surface.

   (e) Pfaffian Differential Forms and Equations.

   (f) Solution of Pfaffian Differential Equations in Three Variables.

   First Order Partial Differential Equations : **[28 Lectures]**

   (a) Curves and surfaces.

   (b) Genesis of First Order Partial Differential Equations.

   (c) Classification of Integrals.

   (d) Linear Equations of the First Order.

   (e) Pfaffian Differential Equations.

   (f) Compatible Systems.

   (g) Charpit's Method.

   (h) Jacobi's Method.

   (i) Integral Surfaces through a given curve.

   (j) Quasi-Linear Equations.

Text Book:

1. Ian Sneddon, Element of Partial Differential Equations, McGraw-Hill Book Company, McGraw-Hill Book Company. Chapter 1 §1 to §6.

2. T. Amaranath, An Elementary Course in Partial Differential Equations, Narosa Publishing, House 2nd Edition, 2003 (Reprint, 2006). Chapter 1 §1 to §10.

Reference Book:

1. Frank Ayres Jr., Differential Equations, McGraw-Hill Book Company, SI Edition (International Edition, 1972)

2. Ravi P. Agarwal and Donal O'Regan, Ordinary and Partial Differential Equations, Springer, First Edition (2009).

3. W.E. Williams, Partial Differential Equations, Clarendon Press, Oxford,(1980).

FMG 3: C Programming
First Term

1. Introductory Concepts: Introduction to computer. Computer Characteristics. Types of Programming Languages. Introduction to C. **[2 Lectures]**
2. C Fundamentals: The character set. Identifier and keywords. Data types. Constants. Variables and arrays. Declarations. Expressions. Statements. Symbolic constants. **[4 Lectures]**
3. Operators and Expressions: Arithmetic operators. Unary operators. Relational and Logical operators. Assignment operators. Conditional Operator. Library functions. **[6 Lectures]**
4. Data Input and Outputs: Preliminaries. Single character input-getchar() function. Single character output-putchar() function. Writing output data-printf function. Formatted input-output. Get and put functions. **[8 Lectures]**
5. Preparing and Running a Program: Planning and writing a C Program. Compiling and Executing the Program. **[2 Lectures]**
6. Control Statements: Preliminaries. The while statement. The do-while statement. The for statement. Nested loops. The if-else statement. The switch statement. The break statement. The continue statement. The comma operator. **[8 Lectures]**
7. Functions: A brief overview. Defining a function. Accessing a function. Passing arguments to a function. Specifying argument data types. Function prototypes. Recursion. **[8 Lectures]**
8. Arrays: Defining an array. Processing an array. Passing arrays to a function. Multidimensional arrays. Arrays and strings. **[10 Lectures]**
Text Book: Programming with C. By Byron S. Gottfried. Schaum's Outline series. Chapters:1,2,3,4,5,6,7,9.
Reference Book: The C Programming Language. By Brian W. Kernighan, Dennis M. Ritchie.

Second term: C programming

1. Program Structures: Storage classes. Automatic variables. External variables. Static variables. **[4 Lectures]**
2. Pointers: Fundamentals. Pointer declarations. Passing pointer to a function. Pointer and one dimensional arrays. Dynamic memory allocation. Operations on pointers. Pointers and multidimensional arrays. Array of pointers. Pointer to function. Passing functions to other functions. More about pointer declarations.
. **[12 Lectures]**
3. Structures and Unions: Defining a structure. Processing a structure. User-defined data types (typedef). Structures and pointers. Passing structure to a function. Self-referential structures. Unions. **[12 Lectures]**
4. Data Files: Opening and closing a data file. Creating a data file. Processing a data file. Unformatted data files. **[10 Lectures]**

5. Low-Level Programming: Bitwise operators. Register variables. Enumerations. Macros. Command line arguments. The C processor. **[10 Lectures]**
Book: Programming with C. By Byron S. Gottfried. Schaum's Outline series. Chapters:8,10,11,12,13,14.
Reference Book: The C Programming Language. By Brian W. Kernighan, Dennis M. Ritchie.