University of Pune  
Board of Studies in Mathematics  
Syllabus for T. Y. B. A. (Mathematics)

<table>
<thead>
<tr>
<th>Course Code</th>
<th>Course Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMG-3</td>
<td>Real Analysis and Lebesgue Integration</td>
</tr>
<tr>
<td>MG-3</td>
<td>Group Theory and Ring Theory</td>
</tr>
<tr>
<td>MS-3</td>
<td>Set Theory, Logic and Metric Spaces</td>
</tr>
<tr>
<td>MS-4</td>
<td>Ordinary and Partial Differential equations</td>
</tr>
<tr>
<td>FMG-3</td>
<td>C-programming</td>
</tr>
</tbody>
</table>
AMG-3: Real Analysis and Lebesgue Integration

First Term: Real Analysis

1. Sequences of real numbers: Definition of sequence and subsequence, Limit of a sequence, convergent sequences, Limit superior and Limit inferior, Cauchy sequences. [10 Lectures]

2. Series of Real numbers: Convergence and divergence of series of real numbers, alternating series, Conditional and absolute convergence of series, test of absolute convergence (Ratio test and Root test), series whose terms form a non-increasing sequence. [10 Lectures]

3. Riemann integral: Sets of measure zero, Definition and existence of a Riemann integral, properties of Riemann integral, Fundamental theorem of integral calculus, Mean value theorems of integral calculus. [14 Lectures]

4. Sequence and series of functions: Pointwise and uniform convergence, sequence of functions, consequences of uniform convergence, convergence and uniform convergence of series of functions, integration and differentiation of series of functions. [14 Lectures]

Text Books:

   Ch. 2 Art. 2.1, 2.9, 2.10.
   Ch. 3 to 3.3, 3.4A, 3.4B, 3.6F, 3.6G, 3.7.
   Ch. 7 Art. 7.1 to 7.4, 7.8 to 7.10.
   Ch. 9 Art. 9.1 to 9.5

Reference Books:


Second term: Lebesgue Integration

1. **Measurable Sets** [12 Lectures]
   (i) Length of open sets and closed sets.
   (ii) Inner and outer measure.
   (iii) Measurable sets.
   (iv) Properties of measurable sets.

2. **Measurable Functions** [12 Lectures]

3. **The Lebesgue integrals** [16 Lectures]
   (i) Definition and example of the Lebesgue integrals for bounded functions.
   (ii) Properties of Lebesgue integrals for bounded measurable functions.
   (iii) The Lebesgue integral for unbounded functions.
   (iv) Some fundamental theorems.

4. **Fourier Series** [8 Lectures]
   (i) Definition and examples of Fourier Series.
   (ii) Formulation of convergence problems.

**Text-Book:**
(Chapter No. 11, 11.1 to 11.8, 12.1, 12.2. Theorem No. 11.1B and 11.1C, 11.8D Statements only).

**Reference Books:**
4. Inder K. Rana, Measure and Integration
MG-3: Group Theory and Ring Theory  
First term: Group Theory  

Groups  

1. Groups : definition and examples.
2. Abelian group, finite group, infinite group.
3. Properties of groups.
4. Order of an element - definition, examples, properties.
5. Examples of groups including \( \mathbb{Z} \), \( \mathbb{Q} \), \( \mathbb{R} \), \( \mathbb{C} \), Klein 4-group, Group of quaternions, \( S^1 \) (= the unit circle in \( \mathbb{C} \)), \( GL_n(\mathbb{R}) \), \( SL_n(\mathbb{R}) \), \( O_n \) (=the group of \( n \times n \) real orthogonal matrices), \( B_n \) (= the group of \( n \times n \) nonsingular upper triangular matrices), and groups of symmetries of plane figures such as \( D_4 \) and \( S_3 \).

Subgroups  

1. Subgroups : definition, necessary and sufficient conditions, examples on finding subgroups of finite groups, union and intersection of subgroups.
2. Subgroup generated by a subset of the group.
3. Cyclic groups : definition, examples of cyclic groups such as \( \mathbb{Z} \) and the group \( \mu_n \) of the \( n \)-th roots of unity, properties :
   (a) Every cyclic group is abelian.
   (b) If \( G = \langle a \rangle \), then \( G = \langle a^{-1} \rangle \).
   (c) Every subgroup of a cyclic group is cyclic.
   (d) Let \( G \) be a cyclic group of order \( n \). Let \( G = \langle a \rangle \). The element \( a^s \in G \) generates a cyclic group of order \( \frac{n}{\gcd(n,s)} \).
   (e) Let \( G = \langle a \rangle \) and \( o(G) = n \). Then \( (a^m) = G \) if and only if \( (m,n) = 1 \).
4. Cosets : definition and properties.
5. Lagrange’s theorem and corollaries.

Permutation Groups  

1. Definition of \( S_n \) and detail discussion of the group \( S_3 \).
2. Cycles and transpositions, even and odd permutations.
3. Order of permutation.
4. Properties: (i) \(|S_n| = n!|\) (ii) \(A_n\) is a subgroup of \(S_n\).

5. Discussion of the group \(A_4\) including converse of Lagrange’s theorem does not hold in \(A_4\).

**Normal Subgroups**

1. Definition.

2. Properties with examples:
   
   (a) If \(G\) is an abelian group, then every subgroup of \(G\) is a normal subgroup.
   
   (b) \(N\) is a normal subgroup of \(G\) if and only if \(gNg^{-1} = N\) for every \(g \in G\).
   
   (c) The subgroup \(N\) of \(G\) is a normal subgroup of \(G\) if and only if every left coset of \(M\) in \(G\) is a right coset of \(N\) in \(G\).
   
   (d) A subgroup \(N\) of \(G\) is a normal subgroup of \(G\) if and only if the product of two right cosets of \(N\) in \(G\) is again a right coset of \(N\) in \(G\).
   
   (e) If \(H\) is a subgroup of index 2 in \(G\) then \(H\) is a normal subgroup of \(G\).
   
   (f) If \(H\) is the only subgroup of \(G\) of a fixed finite order then \(H\) is a normal subgroup of \(G\).

3. Quotient groups and examples.

**Homomorphism and Isomorphism**

1. Homomorphism.

2. Isomorphism: definition, examples, establish isomorphism of two finite groups.

3. Fundamental Theorem of homomorphisms of groups.

4. The group \(\mathbb{Z}/n\mathbb{Z}\) of residue classes (mod \(n\)). Characterization of cyclic groups (as being isomorphic to \(\mathbb{Z}\) or \(\mathbb{Z}/n\mathbb{Z}\) for some \(n \in \mathbb{N}\)).

5. Cayley’s Theorem for finite groups.

6. Classification of groups of order \(\leq 5\).

7. Cauchy’s theorem for Abelian Groups.

**Text book:**


**Reference Books:**

Second term: Ring Theory

1. Definition and properties of Ring, Subring. [5 Lectures]
2. **Integral Domains**: Zero divisors, Cancellation Law, Field, Characteristics of Ring. [5 Lectures]
3. **Ideals and Factor Rings**: Existence of Factor Ring, Prime Ideals, Maximal Ideals. [6 Lectures]
4. **Homomorphism of Rings**: Properties of Ring Homomorphism, Kernel, First isomorphism Theorem for Ring, Prime Fields. The field of Quotients. [8 Lectures]
5. **Polynomial Ring**: Definition. The division Algorithm, Principle Ideal Domain. [6 Lectures]
7. **Divisibility in Integral Domain**: Associates, Irreducible and Primes, Unique Factorization Domains, Ascending chain Condition for PID, PID implies UFD, Euclidean Domains. ED Implies PID, $D$ is UFD implies $D[x]$ is UFD. [10 Lectures]

**Text Book:**

**Chapter Numbers**: 12, 13, 14, 15, 16, 17 and 18.

**Reference Books:**


MS-3: Set Theory, Logic and Metric Spaces
First term: Set Theory and Logic

Sets and Relations: Cantor’s concept of a set, Intuitive set theory, Inclusion, Operations for sets, Algebra of sets, Equivalence relations, Functions, Composition and Inversion of Functions, Operations for collections of sets, Ordering relations, Power sets, Numerical Equivalence of sets. [8 Lectures]

Natural Number sequence:
Induction and Recursion, Cardinal numbers and Cardinality, Cardinal arithmetic, Countable and Uncountable sets, Schroeder-Bernstein Theorem (without proof), Paradoxes of Intuitive set theory, Russell’s Paradox. [12 Lectures]

Logic:
Statement calculus (Sentential connectivities, Truth tables, Validity, Consequence, Applications), Predicate Calculus (Symbolizing every day language, Formulation, Validity, Consequence). [4 Lectures]

Basic Logic:
(Revision) Introduction, proposition, truth table, negation, conjunction and disjunction, Implications, biconditional propositions, converse, contra positive and inverse propositions and precedence of logical operators. [6 Lectures]

Propositional equivalence:
Logical equivalences, Predicates and quantifiers: Introduction, Quantifiers, Binding variables and Negations. [6 Lectures]

Methods of Proof:
Rules of inference, valid arguments, methods of proving theorems; direct proof, proof by contradiction, proof by cases, proofs by equivalence, existence proofs, Uniqueness proofs and counter examples. [12 Lectures]

Text Books:
1. Set Theory and Logic, Robert R. Stoll, Errasia publishers, New Delhi. Sections 1.1 to 1.10, 2.3, 2.4, 2.5

Reference Books:
Second Term: Metric Spaces

1. Chapter 1: Basic Notions. [8 Lectures]
2. Chapter 2: Convergence. [8 Lectures]
3. Chapter 3: Continuity. [8 Lectures]
4. Chapter 4: Compactness. [10 Lectures]
5. Chapter 5: Connectedness. [6 Lectures]
6. Chapter 6: Complete Metric Spaces. [8 Lectures]

Text Book:
Sections: 1.1, 1.2 (except the Sections 1.2.51 to 1.2.65), 2.1, 2.2, 2.3, 2.4, 2.5 and 2.7, 3.1, 3.2 (up to 3.2.32 only), 3.3, 3.4.3.5 (Uniform Continuity to be dropped), 4.1, 4.2, (Proposition 4.2.13 without proof) and 4.3 (Theorem 4.3.24 without proof), 5.1 and 6.1 (Theorems 6.1.1, 6.1.3, 6.1.11, without proofs).

Note: All the problems which are based on normed linear spaces and matrices be dropped.

Reference books:
MS-4: Ordinary and Partial Differential equations

First term: Ordinary Differential Equations

1. What is a Differential Equation?:
   [14 Lectures]
   Introductory Remarks, the nature of solutions, separable equations, first-order linear equations, exact equations, orthogonal trajectories and families of curves, homogeneous equations, integrating factors, reduction of order: (1) dependent variable missing, (2) independent variable missing, electrical circuits.

2. Second-Order Linear Equations:
   [12 Lectures]
   Second-order linear equations with constant coefficients, the method of undetermined coefficients, the method of variation of parameters, the use of a known solution to find another, vibrations and oscillations: (1) undamped simple harmonic motion (2) damped vibrations (3) forced vibrations.

3. Power Series Solutions and Special Functions:
   [12 Lectures]
   Introduction and review of power series, series solutions of first-order differential equations, second-order linear equations, ordinary points, regular singular points, more on regular singular points.

4. System of First-Order Equations:
   [10 Lectures]
   Introductory remarks, linear systems, homogeneous linear systems with constant coefficients.

Text Book: Differential Equations by George F. Simmons, Steven G. Krantz, Tata McGraw-Hill.

Reference Book:
2. Rainville, Bedient: Differential Equations
Second Term: Partial Differential Equations

1. Ordinary Differential Equations in More Than Two Variables
   (a) Surface and Curves in Three Dimensions  [20 Lectures]
   (b) Simultaneous Differential Equations of the First Order and the First Degree in Three Variables.
   (c) Methods of solution of \( \frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} \).
   (d) Orthogonal Trajectories of a System of curves on a Surface.
   (e) Pfaffian Differential Forms and Equations.
   (f) Solution of Pfaffian Differential Equations in Three Variables.

First Order Partial Differential Equations:  [28 Lectures]
   (a) Curves and surfaces.
   (b) Genesis of First Order Partial Differential Equations.
   (c) Classification of Integrals.
   (d) Linear Equations of the First Order.
   (e) Pfaffian Differential Equations.
   (f) Compatible Systems.
   (g) Charpit’s Method.
   (h) Jacobi’s Method.
   (i) Integral Surfaces through a given curve.
   (j) Quasi-Linear Equations.

Text Book:


Reference Book:


FMG 3: C Programming
First Term


5. Preparing and Running a Program: Planning and writing a C Program. Compiling and Executing the Program. [2 Lectures]


Text Book: Programming with C. By Byron S. Gottfried. Schaum’s Outline series. Chapters:1,2,3,4,5,6,7,9.


Second term: C Programming


**Book**: Programming with C. By Byron S. Gottfried. Schaum’s Outline series. Chapters:8,10,11,12,13,14.  