REVISED SYLLABI OF
T.Y.B.A.

(36) (A) STATISTICS (GENERAL) : 1 Paper

(B) STATISTICS (SPECIAL) : 2 Papers

(1 Theory, 1 Practical)

(37) MATHEMATICAL STATISTICS (GENERAL) : 1 Paper

(38) APPLIED STATISTICS (GENERAL) : 1 Paper

(40) STATISTICAL PRE-REQUISITES : 1 Paper
(GENERAL)

(TO BE EFFECTIVE FROM 2010-2011)

UNIVERSITY OF PUNE

Revised Syllabus of

(36) STATISTICS

(General and Special)

Note : (1) A student of the Three-Year B.A. Degree Course offering 'Statistics' at the special level must offer `Mathematical Statistics' as a General level subject in all the three years of the course.

Further students of the three-year B.A. Degree Course are advised not to offer 'Statistics' as the General level subject unless they have offered 'Mathematical Statistics' as a General level subject in all the three years of the course.

(2) A student of three-year B.A. Degree Course offering 'Statistics' will not be allowed to offer 'Applied Statistics' in any of the three years of the course.

(3) A student offering `Statistics' at the Special level must complete all practicals in Practical Paper to the satisfaction of the teacher concerned.

(4) He/She must produce the laboratory journal along with the completion certificate signed by the Head of the Department at the time of Practical Examination.

(5) **Structure of evaluation of practical paper at T.Y.B.A**

(A) Continuous Internal Evaluation	**Marks**
(i) Journal	10
(ii) Viva-voce	10
Total (A)	**20**

(B) Annual practical examination			
Section	**Nature**	**Marks**	**Time**
I	Examination using computer: **Note :** Question is compulsory Q1 : MSEXCEL : Execute the commands and write the same in answer book along with answers	10	Maximum 20 minutes
II	Using Calculator **Note :** Attempt any two of the following **four** questions : Q2 : Q3 : Q4 : Q5 :	60	2 hours 40 minutes
III	Viva-voce	10	10 minutes
	Total (B)	**80**	3 Hours and 10 minutes
	Total of A and B	**100**	---

(6) Duration of the practical examination be extended by 10 minutes to compensate for the loss of time for viva-voce of the candidates.

(7) Batch size should be of maximum 12 students.

(8) To perform and complete the practicals, it is necessary to have computing facility. So there should be sufficient number of computers, UPS and electronic calculator in the laboratory.

(9) In order to acquaint the students with applications of statistical methods in various fields such as industries, agricultural sectors, government institutes, etc. at least one Study Tour for T.Y. B.A. Statistics students must be arranged.

36 (A) STATISTICS (GENERAL)

Title : Design of Experiments and Operations Research

First Term

DESIGN OF EXPERIMENTS

1. Design of Experiments **(20 L)**

Basic terms of design of experiments : Experimental unit, treatment , layout of an experiment.

Basic principles of design of experiments : Replication, randomization and local control.

Choice of size and shape of a plot for uniformity trials, the empirical formula for the variance per unit area of plots.

Analysis of variance (ANOVA): concept and technique.

Completely Randomized Design (CRD) : Application of the principles of design of experiment in CRD, Layout, Model:

(Fixed effect)

$X_{ij} = \mu + \alpha_i + \varepsilon_{ij}$ i= 1,2, …….t, j = 1,2,……., n_i

assumptions and interpretations. Testing normality by pp plot.

Breakup of total sum of squares into components. Estimation of parameters, expected values of mean sums of squares, components of variance, preparation of analysis of variance (ANOVA) table, testing for equality of treatment effects, linear treatment contrast, Hypothesis to be tested

$H_0 : \alpha_1 = \alpha_2 = \ldots = \alpha_t = 0$ and interpretation, comparison of treatment means using box plot technique. Statement of Cochran's theorem.

F test for testing H_0 with justification (independence of chi-squares is to be assumed), test for equality of two specified treatment effects using critical difference (C.D.).

1.6 Randomized Block Design (RBD) : Application of the principles of design of experiments in RBD, layout, model:

$X_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}$ i= 1,2, …….t, j = 1,2,……. b,

Assumptions and interpretations.

Breakup of total sum of squares into components. Estimation of parameters, expected values of mean sums of squares, components

of variance , preparation of analysis of variance table, Hypotheses to be tested $H_{01} : \alpha_1 = \alpha_2 = \alpha_3 = \ldots = \alpha_t = 0$

$H_{02} : \beta_1 = \beta_2 = \beta_3 = \ldots = \beta_b = 0$

F test for testing H_{01} and H_{02} with justification (independence of chi-squares is to be assumed), test for equality of two specified treatment effects using critical difference(C.D.).

1.7 Latin Square Design (LSD): Application of the principles of design of experiments in LSD, layout, Model :

$X_{ij(k)} = \mu + \alpha_i + \beta_j + \gamma_k + \varepsilon_{ij(k)}$ $i = 1,2, \cdots m$, $j = 1,2\cdots,m$, $k=1,2\cdots,m$.

Assumptions and interpretations.

Breakup of total sum of squares into components. Estimation of parameters, expected values of mean sums of squares, components of variance , preparation of analysis of variance table, hypotheses to be tested

$H_{01} : \alpha_1 = \alpha_2 = \cdots = \alpha_m = 0$

$H_{02} : \beta_1 = \beta_2 = \cdots = \beta_m = 0$

$H_{03} : \gamma_1 = \gamma_2 = \cdots = \gamma_m = 0$ and their interpretation.

Justification of F test for H_{01}, H_{02} and H_{03} (independence of chi-squares is to be assumed). Preparation of ANOVA table and F test for H_{01}, H_{02} and H_{03}, testing for equality of two specified treatment effects, comparison of treatment effects using critical difference, linear treatment contrast and testing its significance.

1.8 Analysis of non- normal data using

i) Square root transformation for counts,

ii) $Sin^{-1}(.)$ transformation for proportions,.

iii) Kruskal Wallis H test.

1.9 Identification of real life situations where the above designs are used.

2. Efficiency of Design (5L)

2.1 Concept and definition of efficiency of a design.

2.2 Efficiency of RBD over CRD.

2.3 Efficiency of LSD over CRD and RBD.

3. Split Plot Design (4L)

3.1 General description of a split plot design.

3.2 Layout and model.

3.3 Analysis of variance table for testing significance of main effects and interactions.

4. Factorial Experiments (12L)

4.1 General description of m^n factorial experiment, 2^2 and 2^3 factorial

experiments arranged in RBD.

4.2 Definitions of main effects and interaction effects in 2^2 and 2^3 factorial experiments.

4.3 Yates' procedure, preparation of ANOVA table, test for main effects and interaction effects.

4.4 General idea of confounding in factorial experiments.

4.5 Total confounding (confounding only one interaction), ANOVA table, testing main effects and interaction effects.

4.6 Partial confounding (confounding only one interaction per replicate), ANOVA table, testing main effects and interaction effects.
Construction of layouts in total confounding and partial confounding in 2^2 and 2^3 factorial experiments.

5. Analysis of Covariance (ANOCOVA) with One Concomitant Variable (7L)

5.1 Situations where analysis of covariance is applicable.

5.2 Model for covariance in CRD, RBD. Estimation of parameters (derivations are not expected)

5.3 Preparation of analysis of variance -covariance table, test for ($\beta=0$), test for equality of treatment effects (computational technique only).
Note : For given data, irrespective of the outcome of the test of regression coefficient (β), ANOCOVA should be carried out.

Second Term

OPERATIONS RESEARCH

6. Linear programming (20L)

6.1 Statement of the linear Programming Problem (LPP), Formulation of problem as L.P. problem.
Definition of (i) A slack variable, (ii) A surplus Variable.
L.P. Problem in (i) Canonical form ,(ii) standard form.
Definition of i) a slack variable, ii)a surplus variable. L.P. problem in i) canonical form, ii) standard form.
Definition of i) a solution , ii) a feasible solution, iii) a basic feasible solution, iv) a degenerate and non –generate solution, v) an optimal solution , vi) basic and non- basic variables .

6.2 Solution of L.P.P. by

i) Graphical Method : convex set solution space , unique and non-unique solutions , obtaining an optimal solution, alternate solution, infinite solution, no solution, unbounded solution, sensitivity analysis.

ii) Simplex Method:

a) initial basic feasible solution (IBFS) is readily available : obtaining an IBFS, criteria for deciding whether obtained solution is optimal, criteria for unbounded solution , no solution , more than one solution .

b) IBFS not readily available: introduction of artificial variable, Big-M method, modified objective function, modifications and application of simplex method to L.P.P. with artificial variables.

6.3 Duality Theory: Writing dual of a primal problem, solution of a L.P.P. by using its dual problem.

6.4 Examples and problems.

7. Transportation and assignment problems **(16L)**

7.1 Transportation problem (T.P.), statement of T.P., balanced and unbalanced T.P.

7.2 Methods of obtaining basic feasible solution of T.P. i) North-West corner rule ii) Method of matrix minima (least cost method), iii) Vogel's approximation method (VAM).

7.3 u-v method of obtaing Optimal solution of T.P., uniqueness and non-uniqueness of optimal solutions, degenerate solution.

7.4 Assignment problems: statement of an assignment, balanced and unbalanced problem, relation with T.P., optimal solution of an assignment problem.

7.5 Examples and problems.

8. Sequencing **(6L)**

8.1 Statement of sequencing problem of two machines and n-jobs, three machines and n- jobs (reducible to two machines and n-jobs).

8.2 Calculation of total elapsed time, idle time of a machine, simple numerical problems.

8.3 Examples and problems.

9. Simulation (6L)

9.1 Introduction to simulation, merits, demerits, limitations.

9.2 Pseudo random number generators: Linear congruential generator, mid square method.

9.3 Model sampling from normal distribution (using Box- Muller transformation), uniform and exponential distributions.

9.4 Monte Carlo method of simulation.

9.5 Applications of simulation in various fields.

9.6 Statistical applications of simulation in numerical integration, queuing theory etc.

Note: Verify the solutions using TORA package.

Books Recommended

1. Federer, W.T. : Experimental Design : Oxford and IBH Publishing Co., New Delhi.
2. Cochran W.G. and Cox, C.M. : Experimental Design, John Wiley and Sons, Inc., New York.
3. Montgomery , D.C.: Design and Analysis of Experiments, and sons, Inc., New York.
4. Dass, M.N. and Giri,N.C. : Design and Analysis of Experiments, Wiley Eastern Ltd., New Delhi.
5. Goulden G.H. : Methods of Statistical Analysis, Asia Publishing House Mumbai
6. Kempthhorne, O: Design Analysis of Experiments. Wiley Eastern Ltd., New Delhi.
7. Snedecor, G.W. and Cochran, W.G. : Statistical Methods, Affiliated East – West Press, New Delhi.
8. Goon Gupta,Dasgupta : Fundamentals Of Statistics, Vol.II, The world Press Pvt. Ltd. Calcutta.
9. Gupta S.C. and Kapoor V.K.: Fundamentals of Applied Statistics, S.Chand Sons, New Delhi.
10. C.F. Jeff Wu, Michael Hamda: Experiments, Planning, Analysis and Parameter Design Optimization.
11. G.W. Snedecor , W.G. Cochran : Statistical Methods 8^{th} edition, Eastern Press, Delhi (for 1.8)

12. Miller and Freund : Probability and Statistics for engineers, Pearson Education, Delhi (for 1.8)

13. Gass,E.: Linear programming method and applications,Narosa Publishing House, New Delhi.
14. Taha, R.A.: Operation research, An Introduction, fifth edition, Prentice Hall of India, New Delhi.
15. Saceini,Yaspan,Friedman : Operation Research methods and problems, Willey International Edition.
16. Shrinath.L.S : Linear Programming ,Affiliated East-West Pvt. Ltd , New Delhi.
17. Phillips,D.T, Ravindra , A, Solberg, I.: Operation Research principles and practice , John Willey and sons Inc.
18. Sharma, J.K.: Mathematical Models in Operation Research , Tata McGraw Hill Publishing Company Ltd., New Delhi.
19. Kapoor,V.K.: Operations Research , Sultan Chand and Sons. New Delhi.
20. Gupta, P.K.and Hira , D.S.: Operation Research, S.Chand and company Ltd., New Delhi.

36 (B) STATISTICS (SPECIAL)

S-1 :PAPER I : DISTRIBUTION THEORY

FIRST TERM

1. Multinomial Distribution **(10 L)**

$$P(X_1= x_1, X_2= x_2, \cdots ,X_k=x_k) = \frac{n!\ p_1^{x_1}\ p_2^{x_2} \dots\ p_k^{x_k}}{x_1!x_2!\cdots x_k!},$$

$x_i= 0,1,2\cdots, n;\ i=1,2\cdots,k$

$x_1+x_2+\cdots x_k= n$

$p_1+p_2+\cdots + p_k= 1$

$0< p_i< 1,\ i=1,2\cdots,k$

$= 0$,elsewhere.

Notation : $(X_1, X_2, \cdots ,X_k) \sim MD(n, p_1, \cdots , p_k)$, $\underline{X} \sim MD(n, \underline{p})$

where, $\underline{X} = (X_1, X_2, \cdots , X_k)$, $\underline{P} = (p_1 , p_2 , \dots , p_k)$

1.1 Joint MGF of $(X_1, X_2, \cdots , X_k)$

1.2 Use of MGF to obtain means, variances, covariances, total correlation coefficients, multiple and partial correlation coefficients for k= 3, univariate marginal distributions, distribution of X_i+X_j, Conditional distribution of X_i given $X_i+X_j= r$

1.3 Variance – covariance matrix, rank of variance – covariance matrix and its interpretation.

1.4 Real life situations and applications.

2. Beta distribution **(8L)**

2.1 Beta distribution of first kind

p.d.f. $f(x) = \frac{1}{B(m,n)}$ $x^{m-1} (1-x)^{n-1}$, $0\le x \le 1$, $m,n >0$

$= 0$, elsewhere

Notation : $X \sim B_1 (m,n)$

Nature of probability curve, mean, variance, properties, r^{th} raw moment, harmonic mean.

2.2 Relation with U(0,1), if X and Y are iid $B_1(1,1)$ the probability distributions of

$\frac{1}{X}$, X+Y, X-Y, XY, $\frac{X}{Y}$,

2.3 Beta distribution of second kind

p.d.f. $f(x) = \frac{1}{B(m,n)} \cdot \frac{x^{m-1}}{(1+x)^{m+n}}$, $x \geq 0$, m,n >0

= 0, elsewhere,

Notation : X~ B_2 (m,n)

Nature of probability curve, mean, variance, properties, r^{th} raw moment, harmonic mean.

2.4 Interrelation between B_1(m,n) and B_2 (m,n).

2.5 Distribution of $\frac{X}{Y}$, $\frac{X}{X+Y}$ etc. when X and Y are independent gamma variates.

2.6 Relation between distribution function of B_1(m,n) and binomial distribution.

2.7 Real life situations and applications.

3. Weibull distribution **(6 L)**

p.d.f

$$f(x) = \frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1} \exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\} \qquad x \geq 0, \ \alpha, \beta > 0$$

= 0, elsewhere

Notation : X~ W(α , β).

Probability curve, location parameter, shape parameter, scale parameter, Distribution function, quartiles, mean and variance, coefficient of variation, relationship with gamma and exponential distribution, Hazard rate, IFR,DFR property.

Real life situations and applications.

4. Order Statistics **(10 L)**

Order statistics for a random sample of size n from a continuous distribution, definition, derivation of distribution function and density function of the i-th order statistics $X_{(i)}$ particular cases for i=1 and i=n. Distribution of $X_{(i)}$ for random sample from uniform and exponential distributions.

Derivation of joint p.d.f. of $(X_{(i)}, X_{(j)})$, distribution function of the sample range $X_{(n)}-X_{(1)}$.

Distribution of the sample median.

If $X_1, X_2 \ldots\ldots X_n$ are i.i.d. uniform r.v.s then Corr $(X_{(i)}, X_{(j)})$, distribution of $X_{(n)} - X_{(1)}$ and sample median. Comment on unbiased estimator of θ for $U(0, \theta)$ and exponential(θ) based on order statistics.

5. Chebychev's inequality **(6L)**

5.1 Chebychev's theorem : If g (x) is a non – negative function of r.v. X such that $E[g(X)] < \infty$, then $P[g(X) \geq k] \leq E[g(X)]/k$,where k is positive real number.

5.2 Chebychev's inequality for discrete and continuous distributions in the forms

$P[|X-\mu| \geq k] \leq \frac{\sigma^2}{k^2}$, k>1 and $P[|X-\mu| \geq k\sigma] \leq \frac{1}{k^2}$

where $\mu = E(X)$ and $\sigma^2 = Var(X) < \infty$.

5.3 Applications of Chebychev's inequality in control charts, statistical inference.

6. Central Limit Theorem and Weak Law of Large Numbers **(8 L)**

6.1 Sequence of r.v.s. , convergence of sequence of r.v. in a) probability b) distribution with simple illustrations.

6.2 Statement and proof of the central limit theorem for i.i.d. r.v.s. (proof based on MGF).

6.3 Weak law of large numbers (WLLN).

6.4 Applications of CLT and WLLN.

Second Term

7. Cauchy distribution **(6L)**

7.1 p.d.f. $f(x) = \frac{\lambda}{\pi} \frac{1}{\lambda^2 + (x-\mu)^2}$ $-\infty<x<\infty$, $-\infty<\mu<\infty, \lambda>0$,

= 0, elsewhere

Notation : X~ C (μ,λ).

7.2 Nature of the probability curve.

7.3 Distribution function, quartiles, non – existence of moments, distribution of $aX+b$. Distribution of $\frac{1}{X}$, X^2 for X~ C (0,1)

7.4 Additive property for two independent Cauchy variates (statement only), statement of distribution of the sample mean, comment on limiting distribution of $\bar{X}$.

7.5 Relationship with uniform , Student's t and normal distribution.

7.6 Applications of C(μ,λ).

8. Laplace (double exponential) distribution **(6 L)**

8.1 p.d.f. $f(x) = \frac{\lambda}{2} \exp(-\lambda|x-\mu|)$, $-\infty < x < \infty$, $-\infty<\mu<\infty, \lambda>0$,

= 0, elsewhere

Notation : X~ L (μ,λ).

8.2 Nature of the probability curve.

8.3 Distribution function, quartiles, comment on MLE of μ, λ.

8.4 MGF, CGF, moments and cumulants, β_1, β_2, γ_1, γ_2

8.5 Laplace distribution as the distribution of the difference of two i.i.d. exponential variates with mean $\frac{1}{\lambda}$.

8.6 Applications and real life situations.

9. Lognormal distribution **(8L)**

9.1 p.d.f.

$$f(x) = \frac{1}{(x-a)\sigma\sqrt{2\pi}} \exp\left\{\frac{-1}{2\sigma^2}[\log_e(x-a)-\mu]^2\right\}, \quad a<x, \ -\infty<\mu<\infty, \ \sigma>0,$$

= 0 , elsewhere

Notation : X~ LN (a ,μ,σ^2).

9.2 Nature of the probability curve.

9.3 Moments (r- th moment of X-a), first four moments , β_1 and γ_1

coefficients, quartiles, mode.

9.4 Relation with N (μ,σ^2) distribution.

9.5 Distribution of $\Pi\ X_i$, X_i's independent lognormal variates.

9.6 Applications and real life situations

10. Truncated distributions (8L)

10.1 Truncated distribution, truncation to the right, left and on both sides.

10.2 Binomial distribution B(n,p) left truncated at X=0, (value zero is discarded), its p.m.f. , mean, variance.

10.3 Poisson distribution P(m), left truncated at X=0 ,(value zero is discarded),its p.m.f. mean, variance.

10.4 Normal distribution N ($\mu,\sigma 2$) truncated i) to the left below a ii) to the right above b iii) to the left below a and to the right above b , (a < b) its p.d.f. and derivation of mean and statement (without derivation) of variance in all the three cases.

10.5 Real life situations and applications.

11. Bivariate normal distribution (10L)

11.1 p.d.f. of a bivariate normal distribution.

$$f(x) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp\left\{\frac{-1}{2(1-\rho^2)}\left[\left(\frac{x-\mu_1}{\sigma_1}\right)^2 + \left(\frac{y-\mu_2}{\sigma_2}\right)^2 - 2\rho\left(\frac{x-\mu_1}{\sigma_1}\right)\left(\frac{y-\mu_2}{\sigma_2}\right)\right]\right\}$$

$-\infty<x, y<\infty$,

$-\infty<\mu_1, \mu_2<\infty$,

$\sigma_1, \sigma_2 > 0, -1 < \rho < +1$

$= 0$, elsewhere

Notation : $(X,Y)\sim BN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$, $\underline{X} \sim N_p(\underline{\mu}, \Sigma)$, use of matrix algebra is recommended.

11.2 Nature of surface of p.d.f. ,marginal and conditional distributions, identification of parameters, regression of Y on X and of X on Y, independence and uncorrelated- ness, MGF and moments. Distribution of aX +bY +c , X/Y.

11.3 Applications and real life situations

12. Finite markov chains (10L)

12.1 Definition of a sequence of discrete r.v.s. , Markov property, Markov chain, state space and finite Markov chain (M.C.), one step and n step transition probability, stationary transition probability, stochastic matrix P, one step and n step transition probability matrix

(t.p.m.) Chapman – Kolmogorov equations, t.p.m. of random walk and gambler's ruin problem.

12.2 Applications and real life situations.

Books Recommended

1. H. Cramer : Mathematical Methods of Statistics, Asia Publishing House, Mumbai.
2. Mood, A.M. Graybill, F.Bose, D.C : Introduction to Theory(IIIrd Edition) Mc-Graw Hill Series.
3. B.W. Lindgren : Statistical Theory (IIIrd Edition) Collier Macmillan International Edition, Macmillan Publishing Co.Inc. NewYork.
4. Hogg, R.V. and Craig A.T. : Introduction to Mathematical Statistics (IIIrd Edition), Macmillan Publishing Company, Inc.866 34d Avenue, New York, 10022.
5. Sanjay Arora and Bansi Lal : New Mathematical Statistics (Ist Edition), Satya Prakashan16/17698, New Market, New Delhi,5(1989).
6. S.C. Gupta and V. K. Kapoor : Fundamentals of Mathematical Statistics, Sultan
   Chand and Sons, 88, Daryaganj, New Delhi, 2.
7. V.K. Rohatgi : An Introduction to Probability Theory and Mathematical Statistics, Wiley Eastern Ltd. New Delhi.
8. J. Medhi: Stochastic Processes, Wiley Eastern Ltd…. New Delhi.
9. Hoel, Port and Stone, Introduction to Stochastic Processes, Houghton Miffin.
10. Feller W. : An Introduction of Probability Theory and Its Applications, Vol. I, Wiley Eastern Ltd. Mumbai.
11. Sheldon Ross: A first course in probability (6th edition) : Pearson Education, Delhi.

36 (B) STATISTIC (SPECIAL)

S2 : PAPER II : PRACTICALS

Sr No.	Title of the Experiment
1	Fitting of lognormal distribution.
2	Fitting of lognormal distribution using MS- EXCEL/SPREAD SHEET
3	Fitting of truncated binomial and truncated Poisson distributions
4	Fitting of truncated binomial and truncated Poisson distributions using MS-EXCEL/SPREAD SHEET
5	Applications of truncated binomial, truncated Poisson and truncated normal distributions
6	Model sampling from Laplace and Cauchy distributions (also using MS-Excel / Spread sheet)
7	Applications of bivariate normal distribution
8	Maximum likelihood estimation
9	Testing of hypotheses (UMP test)
10	Construction of confidence interval for proportions, difference of proportions
11	Construction of confidence interval for mean, difference of two means from normal population
12	Construction of confidence interval for variance, ratio of two variances from normal population
13	Nonparametric Tests I (Mann-Whitney. Run test (one sample and two samples), Median test)
14	Nonparametric Tests II (Kolmogorov - Smirnov Test)
15	Sequential probability ratio test for Bernoulli and Poisson distributions (Graphical and Tabular procedure)
16	Sequential probability ratio test for normal and exponential distributions (Graphical and tabular procedure)

17	Solution of LPP by simplex method
18	Solution of LPP by using its dual
19	Transportation problem
20	Analysis of CRD and RBD
21	Analysis of LSD and efficiency
22	Analysis of 2^3 factorial experiments arranged in RBD
23	Analysis of total and partial confounding in 2^3factorial experiments in RBD
24	Analysis of covariance in CRD and RBD

Note :

(1) Computer printouts are to be attached with journal for Experiment Nos. 2,4 and 6.

(2) MS-EXCEL/SPREAD SHEET examination is to be conducted using computer as per note no. (5).

Laboratory Equipments: Laboratory should be well equipped with sufficient number of electronic calculators and computers along with necessary software, printers and UPS.

37) MATHEMATICAL STATISTICS (GENERAL)

STATISTICAL INFERENCE

Note:

1) Mathematical Statistics can be offered only as a general level subject.

2) A student of three Year B.A. Degree Course offering Mathematical Statistics will not be allowed to offer Applied Statistics in any of the three years of the course.

First Term

THEORY OF ESTIMATION

1. Point Estimation (4L)

Notion of a parameter, parameter space , general problem of estimating an unknown parameter by point and interval estimation.

Point Estimation : Definition of an estimator, distinction between estimator and estimate , illustrative examples.

2. Methods of Estimation (12L)

Definition of likelihood as a function of unknown parameter for a random sample from i) discrete distribution ii) continuous distribution, distinction between likelihood function and p.d.f./ p.m.f.

Method of maximum likelihood: Derivation of maximum likelihood estimator (M.L.E.) for parameters of only standard distributions (case of two unknown parameters only for normal distribution). Use of iterative procedure to derive M.L.E. of location parameter μ of Cauchy distribution. Invariance property of M.L.E.

a) M.L.E. of θ in uniform distribution over i) $(0, \theta)$ ii)$(-\theta, \theta)$ iii) $(m\theta, n\theta)$

b) M.L.E. of θ in $f(x ; \theta)= Exp \{-(x- \theta)\}$, $x > \theta$.

Method of Moments : Derivation of moment estimators for standard distributions. Illustrations of situations where M.L.E. and moment estimators are distinct and their comparison using mean square error.

3. Properties of Estimator (20L)

Unbiasedness : Definition of an unbiased estimator, biased estimator, positive and negative bias, illustrations and examples(these should include unbiased and biased estimators for the same parameters). Proofs of the following results regarding unbiased estimators:

a) Two distinct unbiased estimators of $\varphi(\theta)$ give rise to infinitely many estimators.
b) If T is an unbiased estimator of θ , then $\varphi(T)$ is unbiased estimator of $\varphi(\theta)$ provided $\varphi(.)$ is a linear function.

Notion of the Best Linear Unbiased Estimator and uniformly minimum variance unbiased estimator (UMVUE), uniqueness of UMVUE whenever it exists.

Sufficiency: Concept and definition of sufficiency, statement of Neyman's factorization theorem (Proof for discrete probability distribution). Pitmann – Koopman form and sufficient statistic; Exponential family of probability distributions and sufficient statistic.

Proofs of the following properties of sufficient statistics.

i) If T is sufficient for θ, then $\varphi(T)$ is also sufficient for θ provided φ is a one to one and onto function.
ii) If T is sufficient for θ then T is also sufficient for $\varphi(\theta)$.
iii) M.L.E. is a function of sufficient statistic.

Fisher information function: amount of information obtained in a statistic $T = T(x_1,x_2, \ldots, x_n)$. Statement regarding information in $(x_1,x_2,\ldots, x_n)$ and in a sufficient statistics T.

Cramer- Rao Inequality : Statement and proof of Cramer-Rao inequality, Cramer – Rao Lower Bound(CRLB), definition of minimum variance bound unbiased estimator (MVBUE) of $\phi(\theta)$. Proofs of following results:

a) If MVBUE exists for θ then MVBUE exists for $\varphi(\theta)$ where $\varphi(.)$ is a linear function.
b) If T is MVBUE for θ then T is sufficient for θ.

3.6 **Efficiency** : Comparison of variance with CRLB, relative efficiency of T_1 w.r.t. T_2 for (i) unbiased (ii) biased estimators. Efficiency of unbiased estimator T w.r.t. CRLB.

4. Asymptotic Behaviour of an Estimator (6L)

4.1 **Consistency** : Definition , proof of the following theorems:

a. An estimator is consistent if its bias and variance both tend to zero as the sample size tends to infinity.
b. If T is consistent estimator of θ and φ (.) is a continuous function, then φ (T) is a consistent estimator of φ (θ).

4.2 Consistency of M.L.E.(Statement only).

5. Interval Estimation (6L)

5.1 Notion of interval estimation, definition of confidence interval (C.I.), length of C.I., confidence bounds. Definition of pivotal quantity and its use in obtaining confidence intervals.

5.2 Interval estimation for the following cases.

i) Mean (μ) of normal distribution (σ^2 known and σ^2 unknown).
ii) Variance (σ^2) of normal distribution (μ known and μ unknown).
iii) Median, quartiles using order statistics.

Second Term

TESTING OF HYPOTHESES

6. Parametric Tests (15 L)

6.1 (a) Statistical hypothesis, problem of testing of hypothesis. Definition and illustrations of (1) simple hypothesis, (2) composite hypothesis, (3) test of hypothesis, (4) critical region, (5) type I and type II errors. probabilities of type I error and type II error. Problem of controlling the probabilities of errors of two kinds.

(b)Definition and illustrations of (i) level of significance, (ii) observed level of significance (p-value), (iii) size of a test, (iv) power of a test.

6.2 Definition of most powerful(M.P.) level α test of simple null hypothesis against simple alternative. Statement of Neyman - Pearson (N-P) lemma for constructing the most powerful level α test of simple null hypothesis against simple alternative hypothesis. Illustrations of construction of most powerful level α test.

6.3 Power function of a test, power curve, definition of uniformly most powerful (UMP) level α test for one sided alternative. Illustrations.

7. Likelihood ratio test : (9 L)

Notion of likelihood ratio test (LRT), $\lambda(x) = \text{Sup } L(\theta_0|x) / \text{Sup } L(\theta|x)$

Construction of LRT for $H_0 : \theta \in \Theta$ against $H_1: \theta \notin \Theta$ for the mean of normal distribution for i) known σ^2 ii) unknown σ^2 with one tailed and two tailed hypotheses, LRT for variance of normal distribution for i) known μ ii) unknown μ with one tailed and two tailed hypotheses, LRT for parameter of binomial, of exponential etc. two tailed alternative hypotheses, distribution. LRT as a function of sufficient statistics , statement of asymptotic distribution of $-2 \log_e \lambda(x)$.

8. Sequential Tests (9 L)

Sequential test procedure for simple null hypothesis against simple alternative hypothesis and its comparison with fixed sample size N-P test procedure. Definition of Wald's SPRT of strength (α, β). Illustration for standard distributions like Bernoulli, Poisson, Normal and Exponential. SPRT as a function of sufficient statistics. Graphical and tabular procedure for carrying out the test.

9. Nonparametric Tests (15 L)

9.1 Idea of non parametric problems. Distinction between a parametric and a non-parametric problem. Concept of distribution free statistic. One tailed and two tailed test procedure of (i) Sign test, ii) Wilcoxon signed rank test (iii) Mann- Whitney U test, (iv) Run test, one sample and two samples problems.

9.2 Empirical distribution function $S_n(x)$. Properties of $S_n(x)$ as estimator of $F(.)$. Kolmogorov – Smirnov test for completely specified univariate distribution (one sample problem only) with two sided alternative . Comparison with chi-square test.

Books recommended

1. Lindgren, B.W.: Statistical Theory (third edition) collier Macmillan International Edition, Macmillan publishing Co., Inc. New York.

2. Mood, A.M., Graybill, F. and Bose, D.C. : Introduction to the theory of Statistics (third edition) International Student Edition, McGraw Hill Kogakusha Ltd.

3. Hogg, P.V. and Craig, A.J. : Introduction to Mathematical Statistics (fourth edition), Collier Macmillan International Edition, Macmillan Publishing Co. Inc., New York.

4. Siegel, S. : Nonparametric methods for the behavioural sciences, International Student Edition, McGraw Hill Kogakusha Ltd.

5. Hoel, Port, Stone : Introduction to statistical Theory, Houghton Mifflin Company (International) Dolphin Edition.

6. J.D. Gibbons : Non parametric Statistical Interence, McGraw Hill Book Company, New York.

7. Daniel : Applied Nonparametric Statistics, Houghton Mifflin Company, Roston.

8. V.K. Rohatgi : An introduction to probability theory and mathematical statistics, Wiley Eastern Ltd., New Delhi.

9. Kendall and stuart : The advanced Theory of Statistics, Vol 1, Charles and company Ltd., London.

10. Dudeweitz and Mishra : Modern Mathematical Statistic, John Wiley and Sons, Inc., New York.
11. Kale, B.K. : A First Course In parametric Inference.
12. Kunte, S ., Purohit, S.G. and Wanjale, S.K. : Lecture Notes On Nonparametric Tests.
13. B.L. Agarwal : Programmed Statistics: New Age International Publications , Delhi.
14. Sanjay Arora and Bansi Lal : New Mathematical Statistics (I^{st} Edition), Satya Prakashan16/17698, New Market, New Delhi,5(1989).

(38) APPLIED STATISTICS (GENERAL)

APPLICATIONS OF STATISTICS

Note :

(1) Applied Statistics can be offered only as a General Level subject.

(2) A Student of the Three Year B.A. Degree course offering Applied Statistics will not be allowed to offer Mathematical Statistics and/or Statistics in any of the three years of the course.

First term

1. **Continuous type distributions** **(12L)**

1.1 Definition of continuous type of r.v. through p.d.f.,
Definition of distribution function of continuous type r.v.
Statement of properties of distribution function of continuous type r.v.s

1.2 Normal distribution p.d.f.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$$
$$-\infty < x < \infty, \quad -\infty < \mu < \infty, \quad \sigma > 0.$$
$$= 0, \qquad \text{elsewhere}$$

Standard normal distribution, statement of properties of normal distribution, the graph of p.d.f., nature of probability curve. Computation of probabilities.

1.3 Examples and problems

2. **Tests of significance** **(18L)**

2.1 Notion of a statistic as a function $T(X_1, X_2, \ldots, X_n)$ and its illustrations.
2.2 Sampling distribution of $T(X_1, X_2, \ldots, X_n)$. Notion of standard error of a statistic.
2.3 Notion of hypothesis, critical region, level of significance.

2.4 Tests based on normal distribution for

(i) $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0 , \mu < \mu_0 , \mu > \mu_0$
(ii) $H_0 : \mu_1 = \mu_2$ $H_1 : \mu_1 \neq \mu_2 , \ \mu_1 < \mu_2 , \ , \ \mu_1 > \mu_2$
(iii) $H_0 : P = P_0$ against $H_1 : P \neq P_0 , P < P_0 , P > P_0$
(iv) $H_0 : P_1 = P_2$ against $H_1 : P_1 \neq P_2 , P_1 < P_2 , P_1 > P_2$
(v) $H_0 : \rho = \rho_0$ against $H_1 : \rho \neq \rho_0 , \rho < \rho_0 , \rho > \rho_0$

2.5 Examples and problems.

3. Tests based on t, chi-square and F distributions (18L)

3.1 t tests for
(i) $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0 , \mu < \mu_0 , \mu > \mu_0$
(ii) $H_0 : \mu_1 = \mu_2$ $H_1 : \mu_1 \neq \mu_2 , \ \mu_1 < \mu_2 , \ , \ \mu_1 > \mu_2$
(iii) Paired observations

3.2 Tests for $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 \neq \sigma_0^2 , \sigma^2 < \sigma_0^2 , \sigma^2 > \sigma_0^2$

3.3 Chi square test of goodness of fit.

3.4 Chi square test for independence of attributes: Chi square test for independence of 2 x 2 contingency- table (without proof).
Yate's correction not expected.

3.5 Tests for $H_0 : \sigma_1^2 = \sigma_2^2$ against $H_1 : \sigma_1^2 \neq \sigma_2^2 , \sigma_1^2 < \sigma_2^2 , \sigma_1^2 > \sigma_2^2$

Second Term

4. Analysis of variance techniques (12L)

4.1 Concept of analysis of variance.
4.2 One-way and two –way classification : break up of total sum of squares, analysis of variance table, test of hypotheses of (i) equality of several means,
(ii) equality of two means.

4.3 Numerical problems.

5. Non-parametric tests (10L)

5.1 Distinction between a parametric and non-parametric problem.

5.2 Concept of distribution free statistic.

5.3 One tailed and two tailed test procedure of
(a) Sign test, (b) Wilcoxon's signed rank test.

5.4 Test for randomness.

6. Statistical quality control (26L)

6.1 Meaning and purpose of statistical quality control.

6.2 Control chart: Chance and assignable causes of quality variations, statistical basis of control chart (connection with test of hypothesis is NOT expected). Control limits (3-sigma limits only). Criteria for judging lack of control:

(i) One or more points outside the control limits and

(ii) Non-random variations within the control limits : such as a run of seven or more points on either side of the control line, presence of trend or cycle.

6.3 Control charts for variables: Purpose of R-chart and $\overline{X}$ chart, construction of R-chart, $\overline{X}$-chart when standards are not given. Plotting the simple mean and ranges on $\overline{X}$ and R charts respectively. Necessity for plotting R-chart. Revision of R-chart. Drawing conclusion about state of process. Revision of $\overline{X}$ chart. Control limits for future production.

6.4 Control chart for fraction defective (p-chart) only for fixed sample size. Determination of central line, control limits on p-chart, plotting of sample fraction defectives on p-chart. Revision of p-chart, determination of state of control of the process and interpretation of high and low spots on p-chart. Estimation of central line and control limits for future production.

6.5 Control chart for number of defects per unit (c-chart)
Construction of c chart when standards are not given. Plotting of number of defects per unit on c-chart, determination of state of control of the process, revision of control limits for future production.

6.6 Numerical problems based on control charts.

6.7 Identification of real life situations where these charts can be used.

Note: (i) Proof or derivations of results are not expected.

(ii) Stress should be given on numerical problems.

Books Recommended

1. Larson H.J.: Introduction to Probability Theory and Statistical Applications, A Wiley International Edition.
2. Meyer, P. L. Introductory Probability Theory and Statistical Applications, Addison-Wesley Publishing Company.
3. Hoel, P. G.: Introduction. of Mathematical Statistics, John Wiley and Sons Co. New York.
4. Walpole Introduction to Statistics, Macmillan Publishing Co. New York.
5. Lipschutz: Probability and Statistics.
6. Goon, Gupta and Dasgupta : Fundamental's of Statistics, Vol. I, The World Press Pvt. Ltd. Calcutta.
7. New mark, J. : Introduction to Statistics.
8. Miller and Freund: Modern Elementary Statistics.
9. Gupta, S. P. : Statistical Methods, Sultan Chand and Sons, Delhi.
10. Gupta and Kapoor: Fundamentals of Applied Statistics, Sultan Chand and Sons, Delhi.

(40) STATISTICAL PRE-REQUISITES

The course in Statistical Pre-requisites may be offered only by candidates offering one of the social Sciences as their special subject at the B.A. Degree Examination.

The course in Statistical Pre-requisites can not be offered by those who offer any of the Courses in Statistics Groups for their B.A. Examination

First Term

(1) Correlation and Regression
(2) Partial correlation
(3) Multiple correlation
(4) Index numbers
(5) Time series

Second Term

(6) Statistical process control
(7) Acceptance sampling
(8) Data analysis for production function estimation
(9) Economic specialization of the production function
(10) Miscellaneous empirical problems relating to the estimation of production function

Books Recommended

1. Larson$_7$ H.J.: Introduction to Probability Theory and Statistical Applications, A Wiley International Edition.
2. Meyer, P. L. Introductory Probability Theory and Statistical Applications, Addison-Wesley Publishing Company.
3. Hoel, P. G.: Introduction. of Mathematical Statistics, John Wiley and Sons Co. New York.
4. Walpole Introduction to Statistics, Macmillan Publishing Co. New York.
5. Goon, Gupta and Dasgupta : Fundamental's of Statistics, Vol. I, The World Press Pvt. Ltd. Calcutta.
6. New mark, J. : Introduction to Statistics.
7. Miller and Freund: Modern Elementary Statistics.
8. Gupta, S. P. : Statistical Methods, Sultan Chand and Sons, Delhi.
9. Gupta and Kapoor: Fundamentals of Applied Statistics, Sultan Chand and Sons, Delhi.