

UNIVERSITY OF PUNE

Syllabus for F.Y.B.A.

(With effect from Academic Year 2008-2009)

(37) Mathematical Statistics

Paper - Discrete Probability and Probability Distributions

Objectives :

The main objective of this course is to introduce to the students the basic concepts of probability, axiomatic theory of probability, concept of random variable, probability distribution (univariate and bivariate) discrete random variables, expectation and moments of probability distribution.

By the end of the course students are expected to be able :

1. to distinguish between random and non-random experiments.
2. to find the probabilities of events.
3. to obtain a probability distribution of random variable (one or two dimensional) in the given situation, and
4. to apply the standard discrete probability distribution to different real life situations.

(37) Mathematical Statistics

(w.e.f. 2008-2009)

Paper : Discrete Probability and Probability Distributions

Pre requisite : Permutation and Combination theory, Binomial theorem, Algebra of sets.

1. Sample Space and Events (8)

1.1 Experiments/Models, Ideas of deterministic and non-deterministic models.

1.2 Definitions of - (i) Sample space, (ii) Discrete sample space : finite and countably infinite, (iii) Event, (iv) Elementary event, (v) Complement of an event.

1.3 Concepts of occurrence of an event.

1.4 Algebra of events and its representation in set theory notations.

Occurrence of :

(i) at least one of the given events,

(ii) none of the given events,

(iii) all of the given events,

(iv) mutually exclusive events,

(v) mutually exhaustive events,

(vi) exactly one event out of the given events.

1.5 Examples and Problems.

2. Probability (for finite sample space only) (10)

2.1 Equiprobable sample space, probability of an event, certain event, impossible event, classical definition of probability and its limitations, relative frequency approach.

2.2 Non-equiprobable sample space, probability with reference to a finite sample space : probability assignment approach, probability of an event.

2.3 Axioms of probability.

2.4 Probability of union of two events. Theorem of total probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ and its generalization to three events (with proof).

2.5 To prove

(i) $P(A') = 1 - P(A)$, (ii) If $A \subset B, P(A) \leq P(B)$, (iii) $P(\bigcup_{i=1}^k A_i) \leq \sum_{i=1}^k P(A_i)$.
(Boole's inequality).

2.6 Examples and Problems.

3. Conditional Probability and Independence (12)

3.1 Definition of independence of two events

$$P(A \cap B) = P(A) \cdot P(B)$$

3.2 Pairwise independence and mutual independence for three events.

3.3 Definition of conditional probability of an event.

3.4 Multiplication theorem $P(A \cap B) = P(A) \cdot P(B/A)$.

Generalisation to $P(A \cap B \cap C)$.

3.5 Bayes' Theorem (with proof).

3.6 Examples and Problems.

4. Univariate Probability Distributions (defined on Discrete Sample Space) (12)

4.1 Concept and definition of a discrete random variable.

4.2 Probability mass function (p.m.f.) and cumulative distribution function (c.d.f.), $F(\cdot)$ of discrete random variable, properties of (c.d.f.).

4.3 Mode and median of a univariate discrete probability distribution.

4.4 Examples and Problems.

5. Mathematical Expectation (Univariate Random Variable) (12)

5.1 Definition of expectation of a random variable, expectation of a function of a random variable.

5.2 Definitions of mean, variance of univariate probability distribution, effect of change of origin and scale on mean and variance.

5.3 Probability generating function (PGF), Simple properties, mean and variance using PGF.

5.4 Definition of raw, central and factorial moments of univariate probability distributions and their interrelations.

5.5 Examples and Problems.

End of Term

6. Bivariate Probability Distribution (defined on Finite Sample Space) (14)

6.1 Definition of two-dimensional discrete random variable, its joint p.m.f. and its distribution function and their properties.

6.2 Computation of probabilities of events in bivariate probability distribution.

6.3 Concepts of marginal and conditional probability distributions.

6.4 Independence of two discrete random variables.

6.5 Examples and Problems.

7. Mathematical Expectation (Bivariate Random Variable) (14)

7.1 Definition.

7.2 Theorems on expectations of sum and product of two jointly distributed random variables.

7.3 Conditional expectation.

7.4 Definitions of conditional mean and conditional variance.

7.5 Definition of raw and central moments.

7.6 Definition of covariance, correlation coefficient (ρ), independence and uncorrelatedness of two variables.

7.7 Variance of linear combination of variables.

7.8 Examples and Problems.

8. Some Standard Discrete Probability Distribution (28)

8.1 Uniform discrete distribution on integers 1 to n : - p.m.f., c.d.f., mean, variance, real life situations, comments of mode and median.

8.2 Bernoulli Distribution : p.m.f., mean variance, moments, distribution of sum of independent identically distributed Bernoulli variables.

8.3 Binomial Distribution : p.m.f.

$$P(x) = \begin{cases} \binom{n}{x} p^x q^{n-x} & x = 0, 1, \dots, n, \quad 0 < p < 1, \quad q = 1 - p \\ 0, & \text{otherwise} \end{cases}$$

Notation : $X \sim B(n, p)$.

Recurrence relation for successive probabilities, computation of probabilities of different events, computation of median for given parameters, mode of the distribution,

mean, variance, moments, skewness (comments when $p = 0.5, p > 0.5, p < 0.5$), P.G.F. additive property of binomial variables, conditional distribution of X given $X + Y$, where X and Y are independent, $B(n_1, p)$ and $B(n_2, p)$ variables.

8.4 Hypergeometric Distribution : p.m.f.,

$$p(x) = \begin{cases} \frac{\binom{M}{x} \binom{N-M}{n-x}}{\binom{N}{n}} & , \quad x = a, a + 1, \dots, b \\ 0, & \text{otherwise} \end{cases}$$

where $a = \max(0, n - N + M), b = \min(n, M)$

Notation : $X \sim H(N, M, n)$.

Computation of probability, situations where this distribution is applicable, binomial approximation to hypergeometric probabilities, mean and variance of the distribution.

8.5 Poisson Distribution : p.m.f. $p(x) = e^{-m} \frac{m^x}{x!}, x = 0, 1, 2, \dots; m > 0$. State the mean, variance, additive property (no derivation). Derivation of Poisson distribution as a limiting case of binomial distribution.

8.6 Example and Problems.

Reference Books

1. Hogg, R. V. and Craig R. G. : Introduction to Mathematical Statistics, Ed. 4. (1989), MacMillan Publishing Co., New York.
2. Hoel, P. G. : Introduction to Mathematical Statistics (1962), John Wiley and Sons, New York.
3. Feller, W. : Introduction to Probability Theory and Its Applications, Vol. I (1963), Asian Publishing House, Bombay.
4. Mood, A. M. and Graybill, F. A. and Boes D.C. E. : Introduction to Theory of Statistics, Ed. 3 (1974), McGraw Hill and Kagakusha Ltd. London.
5. Mayer, P. N. : Introduction to Probability and Statistical Applications, Addison Wesley Publishing Co., Massachusetts).

6. Gupta and Kapoor : Fundamentals of Mathematical Statistics, Sultan Chand and Sons, New Delhi.
7. Ross : Probability theory, Pearson Publishers.
8. M. B. Kulkarni and S. B. Ghatpande : Discrete Probability and Probability Distributions, SIPF Academy, Nashik.
9. B. L. Agarwal : Programmed Statistics, New Age International Publishers, New Delhi.
10. K. V. S. Sarma : Statistics Made Simple : Do it yourself on PC. Prentice Hall, New Delhi.

Reference Websites :

1. www.stats.unipune.ernet.in (100 Data sets for Statistics Educagtion by Dr Anil P. Gore, Dr. Mrs. S. A. Paranjpe and Madhav B. Kulkarni available in ISPS folder).
2. www.freestatistics.tk
3. www.psychstat.smsu.edu/sbk00.htm
4. www.bmj.bmjournals.com/collections/statsbk/index.shtml
5. www.statweb.calpoly.edu/bchance/stat-stuff.html
6. www.amstat.org/publications/jse/jse-data-archive.html
7. www.statpages.org (Webpages that perform statistical calculations)
8. www.amstat.org/publications/chance (Chance magazine)
9. www.statsci.org/datasets.html (Data sets)
10. www.math.uah.edu/stat (Virtual laboratories in Statistics)
11. www.amstat.org/publications/stats (STATS : the magazine for students of Statistics)
12. www.stat.ucla.edu/cases (Case studies in Statistics)