

M. A./M. Sc. Mathematics / 1

Publisher's Note

The University of Pune has great pleasure in publishing Revised syllabus for the **M.A./M.Sc. Mathematics** Examination under the Faculty of Arts and Fine Arts/Science.

It is hoped that this syllabus will be most useful to the students of this course.

On behalf of the University, I thank experts and authorities of the University for their keen interest and whole-hearted co-operation in bringing out this publication.

University of Pune
Ganeshkhind, Pune-411 007.

Dr. D. D. Deshmukh
Registrar

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University of Pune
Syllabus of M. A./M.Sc. Mathematics Courses

Semester I

MT	101	Advanced Calculus
MT	102	Foundations of Analysis
MT	103	Complex Analysis
MT	104	Algebra
MT	105	Differential Equations

Semester II

MT	201	Topology
MT	202	Measure and Integration
MT	203	Functional Analysis
MT	204	Linear Algebra
MT	205	Mechanics

Semester III

University Courses (Exactly Three)

MT	301	Algebraic Topology
MT	302	Boundary Value Problems
MT	303	Rings and Modules
MT	304	Graph Theory
MT	305	Numerical Analysis
MT	306	Advanced Complex Analysis
MT	307	Field Theory
MT	308	Computer Science I

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Departmental Courses (Exactly Two)

MT	311	Topics in Discrete Mathematics-I
MT	312	Topics in Applied Mathematics-I
MT	313	Topics in Graph Theory-I
MT	314	Topics in Algebra-I
MT	315	Topics in Complex Analysis
MT	316	Topics in Ring Theory-I
MT	317	Topics in Boundary Value Problems-I
MT	318	Topics in Functional Analysis-I
MT	319	Topics in Analysis-I
MT	320	Topics in Manifolds-I
MT	321	Topics in Lattice Theory-I
MT	322	Topics in Analytic Function Theory
MT	323	Topics in Relativity-I
MT	324	Topics in Number Theory-I
MT	325	Topics in Numerical Analysis-I
MT	326	Topics in Approximation Theory and Interpolation-I
MT	327	Topics in Operator Theory-I
MT	328	Topics in Computer Science-I
MT	329	Topics in Algebraic Topology-I
MT	330	Topics in Geometry-I

Semester IV

University Courses (Exactly Three)

MT	401	Commutative Algebra
MT	402	Hydrodynamics
MT	403	Lattice Theory
MT	404	Combinatorics
MT	405	Several Complex Variables
MT	406	Banach Algebras and Spectral Theory

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MT	407	Mathematical Analysis
MT	408	Computer Science II

Departmental Courses (Exactly Two)

MT	411	Topics in Discrete Mathematics-II
MT	412	Topics in Applied Mathematics-II
MT	413	Topics in Graph Theory-II
MT	414	Topics in Algebra-II
MT	415	Topics in Complex Analysis-II
MT	416	Topics in Ring Theory-II
MT	417	Topics in Boundary Value Problems-II
MT	418	Topics in Functional Analysis-II
MT	419	Topics in Analysis-II
MT	420	Topics in Manifolds-II
MT	421	Topics in Lattice Theory-II
MT	422	Topics in Analytic Function Theory-II
MT	423	Topics in Relativity-II
MT	424	Topics in Number Theory-II
MT	425	Topics in Numerical Analysis-II
MT	426	Topics in Approximation Theory and Interpolation-II
MT	427	Topics in Operator Theory-II
MT	428	Topics in Computer Science-II
MT	429	Topics in Galoi Field Theory
MT	430	Topics in Geometry-II

1. In each of the IIIrd Semester and IVth Semester, a student must select exactly three University Courses and two Departmental courses. A course cannot be started unless there are minimum five students for that particular course.

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2. Departmental courses may be proposed by the Department only and the approval of the Board of Studies (BOS) in Mathematics should be taken.
3. It shall be obligatory for all the concerned to get the syllabi of Departmental course approved by the BOS in Mathematics prior to starting of the courses.
4. For external students all the courses shall be University courses.

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MT 101 : Advanced Calculus

Derivative of a scalar field with respect to a vector, Directional derivatives, total derivative, gradient of a scalar field, Derivative of a vector field, Matrix form of the chain rule.

Paths and line integrals, the concept of work as a line integral. Independence of path. The first and the second fundamental theorems of calculus for line integral. Necessary condition for a vector field to be a gradient.

Double integrals, Applications to area and volume, Green's Theorem in the plane, change of variables in a double integral. Transformation formula, change of variables in an n-fold integrals.

The Fundamental vector product, area of a parametric surface, surface integrals. The theorem of Stokes, the curl and divergence of a vector field. Gauss divergence theorem. Applications of the divergence theorem.

Prescribed Text Book

T.M. Apostol : Calculus Vol. II (2nd edition).

Chapter 1 : Sections 8.1 to 8.22.

Chapter 2 : Sections 10.1 to 10.11 and 10.14 to 10.16.

Chapter 3 : Sections 11.1 to 11.15, 11.19 to 11.22 and 11.26 to 11.34.

Chapter 4 : Sections 12.1 to 12.15, 12.18 to 12.21.

Convention : The vector notations denoted by bold letters in the text may be denoted by letters with arrows above (i.e. \vec{x} , \vec{a} etc.).

Reference Books

1. T.M. Apostol : Mathematical Analysis (Old edition)
2. W. Rudin : Principles of Mathematical Analysis.
3. Devinata : Advanced Calculus.

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MT 102 : Foundations of Analysis

Cardinality of a set, countable and uncountable sets, Schroder-Bernstein theorem. Relation between cardinality of a set and cardinality of its power set.

The metric spaces \mathbb{R} and \mathbb{C} , The metric space $C[0,1]$ of bounded continuous functions on the closed and bounded interval $[0,1]$. Open and closed sets in a metric space. Cantor's set, convergence, completeness in metric spaces.

Cantor's intersection theorem. Continuous mapping in metric spaces. Baire's category theorem. The spaces $C(X, \mathbb{R})$, $C(X, \mathbb{C})$ and l^p . The Heine-Borel theorem, Compactness for metric spaces, Bolzano-Weierstrass property, Lebesgue's covering lemma, Ascoli's theorem. Hausdorff spaces. Connectedness in \mathbb{R} , \mathbb{C} . The Weierstrass Approximation theorem. The Stone-Weierstrass theorem.

Prescribed Text Book

G.F.Simmons : Introduction to Topology and Modern Analysis [McGraw-Hill International Edition (1963)].

Chapter I (Sections 5, 6, 7).

Chapter II (Sections 9, 10, 11, 12, 13, 14, 15).

Chapter IV (Sections 21, 24, 25).

Chapter V (Sections 26).

Chapter VI (Section 31).

Chapter VII (Sections 35, 36).

Reference Books

1. J. F. Randolph : Basic Real and Abstract Analysis.
2. Dieudonne : Modern Analysis.
3. K. D. Joshi : Introduction to General Topology.

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MT 103 : Complex Analysis

1. Inequalities involving complex numbers. The spherical representation of the complex plane C . (Chapter 1 - 1.5, 2.4).
2. Limits and Continuity (Revision).
Analytic Functions, Necessary and Sufficient conditions (Cauchy-Riemann equations). Polynomials, Lucas's theorem, Rational Functions.
(Chapter 2 - 1.1, 1.2, 1.3, 1.4).
3. Connectedness, compactness in C , Heine-Borel Property in C , Bolzano-Weierstrass theorem for C , Continuous Functions on compact subsets of C , Arcs and closed curves, Analytic Functions in regions, Conformal mapping, Linear transformation, The linear group, The cross ratio, Symmetry.
(Chapter 3 - 1.3, 1.4, 1.5, 2.1, 2.2, 2.3 3.1, 3.2, 3.3).
4. Line integrals, Rectifiable arcs, Line integrals as functions of arcs, Cauchy's theorem for a rectangle, Cauchy's theorem in a disk, The index of a point with respect to a closed curve. Cauchy's integral formula. Higher derivatives. Morera's theorem, Liouville's theorem, Classical theorem of Weierstrass concerning behaviour of a function in the neighbourhood of an essential singularity, The local mapping, open mapping theorem The maximum principle, Schwarz's lemma, The residue theorem, The argument principle, Rouché's theorem, Evaluation of definite integrals.
(Chapter 4 - 1.1 to 1.5, 2.1, 2.2, 2.3, 3.1 to 3.4, 4.1 to 4.5, 5.1 to 5.3).
5. Power series expansions, The Weierstrass theorem, The Taylor series, The Laurent series.
(Chapter 5 - 1.1, 1.2, 1.3).

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Prescribed Text

Complex Analysis - Lars V. Ahlfors (Third Edition)
(McGraw-Hill International Editions).

Reference Books

1. J. B. Conway : Functions of one complex variable.
2. S. Lang : Complex Analysis.
3. H. Silverman : Complex Variables.
4. Moore and Haddock : Complex Analysis.

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MT 104 : Algebra

Homomorphism of groups, The symmetric group S_n , The commutator subgroup, The normalizer, Cyclic groups, Lagrange's theorem, Euler-Fermat theorem, Conjugacy, Class equations, Normal subgroups, Quotient groups, Homomorphism theorems, Group actions, Cayley's theorem.

Direct products (two factors), Direct product (general case), Finite abelian groups, Sylow theorems.

Theorems of Schreier and Jordan - Holder, Solvable groups, Nilpotent groups.

Prescribed Text Book

I. S. Luthar and I. B. S. Passi : Algebra, Volume 1 (Narosa Publishing House).

Reference Books

1. N. Jacobson : Basic Algebra, Vol. 1.
2. I. N. Herstein : Topics in Algebra.
3. M. Artin : Algebra (Prentice Hall).
4. N. S. Gopalkrishnan : University Algebra (Wiley Extern).

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MT 105 : Differential Equations

Review : General remarks on solutions of differential equations, Families of curves, Orthogonal trajectories.

1. Second Order Linear Equations.
The general solution of the homogeneous equations, Use of a known solution to find another solution, Homogeneous equations with constant coefficients. The method of undetermined coefficients. The method of variation of parameters.
2. Qualitative properties of solutions of Ordinary differential equations of order two Sturm Separation theorem. Normal form. Standard form, Sturm's comparison theorem.
3. Power Series Solutions and special functions. Review of power series, Series solutions of first order equations, Second order linear equations. Ordinary points, Regular singular point, Indicial equations. Gauss Hypergeometric equation. The point at infinity. Legendre polynomials, properties of Legendre polynomials, Bessel Functions, Properties of Bessel Functions.
4. Systems of First Order Equations
General remarks on systems, Linear systems, Homogeneous linear system with constant coefficients.
5. The Existence and Uniqueness of solutions.
The method of successive approximations, Picard's theorem, Existence and uniqueness of Second order initial value problems.

Prescribed Text Book

G. F. Simmons : Differential Equations with applications and Historical notes (Tata - McGraw Hill).

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Reference Books

1. G. Birkhoff and G. C. Rota : Ordinary Differential Equations (John Wiley and Sons).
2. E. A. Coddington : Ordinary Differential Equations (Prentice Hall of India)
3. R. P. Agarwal and R. C. Gupta : Essentials of Ordinary Differential Equations (McGraw Hill).

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MT 201 : Topology

Arbitrary Cartesian Products, Topological spaces, Basis for a topology, finite product topology, Subspace Topology, Closed sets, limit points, Hausdorff Spaces, Continuous Functions, Homomorphisms. The Pasting lemma, arbitrary Product topology, The quotient topology, Connected Spaces, Path connected spaces, Components and path components, Local Connectedness. Compact Spaces, The tube lemma, Limit point compactness, Local compactness, One point compactification. The countability axioms. Separation Axioms. Urysohn's lemma, Tietze Extension theorem, Metrization Theorem, Tychonoff's Theorem.

Prescribed Book

J. R. Munkres : Topology A first course (Prentice Hall, India), 1975. Sections : 1.5, 2.1, 2.2, 2.4-2.8, 2.11, 3.1, 3.2, (Theorems 2.1 and 2.3 omitted, Example 6 omitted), 3.3, 3.4, 3.5, 3.7 (Theorems 7.3 and 7.4 omitted) 3.8, 4.1, 4.2 (Theorems 2.6 and Example 2 omitted) 4.3, 5.1, 7.7.

Reference Books

1. G. F. Simmons : An Introduction to Topology and Modern Analysis (Mc Graw Hill).
2. S. Willard : General Topology (Addison Wesley).
3. J. L. Kelly : General Topology.
4. K. D. Joshi : Introduction to General Topology.

MT 202 : Measure and Integration

Drawbacks of Riemann Integral (without proofs). A function theoretic view of the Riemann integral. Lebesgue's recipe. Semialgebra and algebra of sets. Extending a set function from a semi-algebra to the algebra generated by it. Countably additive set functions on intervals; countably additive set functions on algebras. Approximating the length of a set $A \subseteq \mathbb{R}$: the induced outer measure. Choosing nice sets: measurable sets. The σ -algebras and extending a measure from an algebra to the σ -algebra generated by it. Uniqueness of extension. Measure Space, Complete measure space and Completion of measure space.

Unique extension of the length function: The Lebesgue measure. Relation of the Lebesgue measurable sets with topologically nice sets of \mathbb{R} . Properties of the Lebesgue measure with respect to the group structure on \mathbb{R} . Uniqueness of Lebesgue measure.

Integration: The integral of a non-negative simple function, extending the integral beyond non-negative simple functions, intrinsic characterization of the Class II⁺: non-negative measurable functions. Integrable functions. The Lebesgue integral and its relation with the Riemann integral L , $[a,b]$ as completion of $\mathbb{R} [a,b]$.

Fundamental Theorem of Integral Calculus for Lebesgue Integrals: Absolutely continuous function, Differentiability of monotone functions. Fundamental theorem of calculus and its applications. Measure and integration on Product Spaces: Product of two measure spaces. Integration on Product Spaces, Fubini's theorems. Modes of Integration of Complex Valued Functions. Convergence: Pointwise, almost everywhere, uniform and almost uniform. Convergence in measure L_p -

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Spaces and convergence in L_p . Absolutely continuous measures, and Random-Nikodym theorem. Computation of the Random-Nikodym derivative. Change of variable formulas.

Prescribed Book

Inder K. Rana : An Introduction to Measure and Integration (Narosa).

Sections : 1.5, 2.1, 2.2, 3.2, 3.3, 3.5, 3.6, 3.7, 3.8, 3.9, 3.10, 3.11, 4.1, 4.2, 4.3, 4.4, 5.1, 5.2, 5.3, 5.4, 5.5, 5.6, 6.1, 6.2, 6.3, 7.1, 7.2, 7.3, 8.1, 8.2, 8.3, 8.4, 8.5, 9.1, 9.2 and 9.3.

Reference Books

1. S. K. Berberian : Measure and Integration.
2. de Barra G. : Measure Theory and Integration.
3. H. L. Roydon : Real Analysis.

MT 203 : Functional Analysis

1. Banach Spaces : Definition and examples. Quotient Spaces. Convexity of the closed unit sphere of a Banach Space. Examples of non-normal spaces. Holder's inequality. Minkowski's inequality. Linear transformations on a normed linear space and characterization of continuity of such transformations. The set script $B(N, N')$ of all bounded linear transformations of a normed linear N into normed linear space N' . space N' . Equivalent norms. All norms on a finite dimensional normed linear spaces are equivalent. The conjugate space N^* . The Space $(L^p)^*$. Hahn-Banach theorems. The natural imbedding of N into N^{**} . Reflexive spaces. Compact Hausdorff nature of the closed unit sphere of a normed linear space with respect to weak* topology. For a Banach space B , B is reflexive iff B^* is reflexive. The open mapping theorem. Projections on a Banach Space. The closed graph theorem. The uniform boundedness theorem. The conjugate T^* of an operator T and related results.
2. Hilbert Spaces : Definition and Examples, Schwarz's inequality. Parallelogram Law. A closed convex subset of a Hilbert Space contains a unique vector of the smallest norm. Orthogonal decomposition of a Hilbert space. Orthogonal sets in a Hilbert space. Bessel's inequality. Characterization of complete orthonormal set. Gram-Schmidt orthogonalization process. The conjugate space H^* of a Hilbert space H . Representation of a functional f as $f(x) = (x, y)$ with y unique. The Hilbert space H^* . Interpretation of T^* as an operator on H . The adjoint operator $\underline{T} - \underline{T}^*$ on $B(H)$. self-adjoint operators, Positive operators. Normal operators. Unitary

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operators and their properties, Projections on a Hilbert space. Invariant subspace. Orthogonality of rejections.

3. Eigen values and eigen space of an operator on a Hilbert Space. Spectrum of an operator on a finite dimensional Hilbert Space. Finite dimensional spectral theorem.
4. Fixed Point. Every contraction on a complete metric space has a unique fixed point. Picard's theorem for existence and uniqueness of solution of a differential equation of order one, degree one.

Prescribed Text Book

G. F. Simmons : Topology and Modern Analysis (McGraw-Hill International Edition).
Chapters 9, 10, 11 and Appendix-one.

Reference Books

1. G. Backman and L. Narici : Functional Analysis (Academic).
2. Liusternik and Soboler : The Elements of Functional Analysis (Friedrick Ungart).
3. B. Limaye : Functional Analysis (Wiley Eastern).

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MT 204 : Linear Algebra

0. Revision : Vector Spaces, Linear transformations.
1. The Algebra of Linear transformation, Rank of a Linear transformation, Characteristic Roots, Relation between characteristic roots and characteristic vectors.
2. Canonical Forms : Triangular Forms, Nilpotent transformations, Trace and Transpose. Determinants.
3. Hermitian, Unitary and Normal transformations, Real quadratic forms.

Prescribed Text Book

I. N. Herstein : Topics in Algebra, Wiley Eastern Ltd. (Vikas Publishing House Pvt. Ltd.) 1988. IInd Edition. Chapter VI : Sections 1 to 6, Section 8, Section 9, Section 10 and Section 11.

Reference Books

1. J. S. Golan : Foundations of Linear Algebra, Kluwer Academic Publishers (1995).
2. Henry Helson : Linear Algebra, Hindustan Book Agency (1994).
3. A. G. Hamilton : Linear Algebra, Cambridge University Press (1989).
4. K. Hoffman & Ray Kunze : Linear Algebra, Prentice Hall, India (1972).

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MT 205 : Mechanics

1. Constrained Motion : Constraints, their classification, examples, principle of virtual work, D’Hembert’s principle, Lagrange’s equations.
Sections : 1.1, 1.2, 1.3, 1.4, 1.5, 1.7.
2. Lagrangian Formulation : Degrees of freedom, Generalised coordinates, Lagrange’s equations of motion of the second kind, properties of kinetic energy function. Generalised momenta and energy, Gauge function, cyclic or ignorable coordinates, integral of motion, concept of symmetry, invariance under Galilean transformations, Lagrangian for free particle motion.
Sections : 2.1 to 2.16 [except 2.8 & 2.11].
3. Central Force : Properties of the central force, two-body central force problem, stability of orbits, conditions for closure, integrable power laws of the central force, derivation of force laws.
Sections : 4.1, 4.2, 4.3, 4.4, 4.5, 4.6.
4. Hamilton’s equations of motion : Legendre transformation, Hamilton’s equs of motion, Routhian, Configuration Space, Phase, State Space examples.
Sections : 5.1, 5.2, 5.3, 5.4, 5.5.
5. Canonical Transformations : Generating Functions, Properties of canonical transformations, examples, Liouville’s theorem, Area conservation property of Hamiltonian flows.
Sections : 8.1, 8.2, 8.3, 8.4, 8.5, 8.6, 8.7.
6. The Poisson Bracket : Definition, Some identities, elementary Pbs. Poisson’s theorem, Jacobi-Poisson theorem on PBS. Invariance of PB under canonical transformations.
Sections : 9.1, 9.2, 9.3, 9.4, 9.5, 9.6.

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Text Books

Rana and Joag : Classical Mechanics : Tata McGraw Hill Publishing Company Limited.

Recommended Books

1. Goldstein : Classical Mechanics (Ed. 2), Narrosa Publishing House.
2. Landau and Lifshitz : Mechanics (Pergamon Press).
3. Greenwood : Classical Dynamics (Prentice-Hall).

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MT 301 : Algebraic Topology

1. Fundamental group of a space, the effect of the continuous mapping on the fundamental group, the fundamental group of a circle is infinite cyclic. The Brouwer Fixed point theorem in dimension.
2. The weak product of abelian groups, free abelian group, free product of groups, the presentation of groups by generators and relations.
3. Seifert and Van Kampen theorem on the fundamental group of the union of two spaces, applications to arcwise connected spaces.
4. Covering spaces, lifting of paths to a covering space, the fundamental group of covering space, lifting of arbitrary maps to covering spaces, the action of the group $\mathcal{P}(X, x)$ on the set $P^{-1}(x)$, regular covering spaces and quotient spaces. The Borsuk-Ulam theorem for the 2-spheres, the existence theorem for covering spaces.

Prescribed Text Book

W. S. Massey : Algebraic Topology - An Introduction (Springer-Verlag).

Chapter 2 : Sections 1 to 6, Chapter 3 : Sections 1 to 6, Chapter 4 : Sections 1 to 4, Chapter 5 : Sections 1 to 10.

Reference Books

- 1 J. R. Munkres : Topology - A First Course (Prentice-Hall).
- 2 W. Fulton : Algebraic Topology - A first course (Springer-Verlag).
- 3 N. Singer and A. Thorpe : Lecture Notes on Elementary Topology and Geometry (Springer-Verlag)

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MT 302 : Boundary Value Problems

1. Definition of Boundary value problems, the heat equation, wave equation, Laplace's equation, the Fourier method, Linear Operators, Principle of Superposition, series of solutions, uniform convergence (Weierstrass M-test), separation of variables, non-homogeneous conditions, Sturm-Liouville problems, formal and rigorous solutions, the vibrating string.
2. Orthogonal sets of functions, Generalized Fourier series, Best approximation in the mean. Convergence in the mean, the orthonormal trigonometric functions, other types of orthogonality.
3. Sturm-Liouville Problems and applications, orthogonality and uniqueness of eigenfunctions, methods of solutions, surface heat transfer, other boundary value problems.
4. Bessel Function J_n recurrence relations, the zeros of $J_0(x)$ and related functions, Fourier-Bessel series, Temperatures in a long cylinder.
5. Legendre Polynomials, orthogonality of Legendre polynomials, Legendre series, Dirichlet problems in spherical regions.

Prescribed Text Book

R. V. Churchill and J. Brown : Fourier series and Boundary value problems (4th edition).

Reference Books

1. D. L. Powers : Boundary value problems, 2nd edition, (Academic Press) (1971).
2. Kaplan : Advanced Calculus (Addison-Wesley Publishing Company) (1984).
3. E. D. Rainville : Special Functions (Chelsea Publishing Company) New York (1971).

MT 303 : Rings and Modules

1. Rings, zero-divisors, nilpotent element, idempotent element, Rings of continuous functions, Matrix Rings, Polynomial rings, Power series rings. Laurent rings, Boolean rings, Rings of integers mod. n , Rings of Gaussian integers, Ring of quaternions, Ring of Piadic complex entire functions, Direct products, Several variables, opposite rings, Characteristic of a ring.
2. Ideals, Maximal Ideals, Generators, Properties of Ideals, Algebra of Ideals, Quotient rings, Ideals in Quotient rings.
3. Homomorphisms of rings, Fundamental Theorems of homomorphism. Field of fractions, Prime fields.
4. Euclidean and Principle ideal domains, Unique factorization domains, Eisenstein's criterion.
5. Modules : Definitions and examples. Direct sums, Free modules, Quotient modules, Homomorphisms, Simple modules, Modules over PID's.

Prescribed Text Book

C. Musili : Introduction to Rings and Modules (Narosa Publishing House) (1994) (2nd edition)

Chapters : I, II, III, IV, and V.

Sections : 1.1 - 1.12, 2.1-2.7 (Section 2.8 omitted), 3.1 - 3.5, 4.1 - 4.6

Reference Books

1. M. Nagata : Theory of Fields
2. S. Lang : Algebra
3. M. Artin : Algebra

MT 304 : Graph Theory

1. Graphs : Definitions and examples, Graphs as models. Subgraphs, walks, paths and cycles, Connectedness, Matrix representation of graphs, Operations on graphs, connectedness algorithm.
2. Trees and connectivity : Definition and simple properties, Bridge, spanning trees, Caley's theorem. Connector problems. Kruskal's Algorithm, Prim Algorithm. Shortest path problems. The Breadth First Search Algorithm. The Back-tracking Algorithm. Dijkstra's Algorithm. Cut vertices, Connectivity.
3. Euler Tours and Hamiltonian Cycles : Euler Tours, Konigsberg Seven bridges problem Eulerian graphs. Fleury's Algorithm, Hierholzer's Algorithm. The Chinese postman Problem. Hamiltonian graphs. Dirac theorem. Closure of a graph. Bondy and Chavatal Theorem. The travelling salesman problem. The two-optimal algorithm. The closest Insertion Algorithm.
4. Matchings : Matching and Augmenting paths, Berge theorem. The Marriage problem. The Personnel Assignment problem. The matching algorithm for bipartite graphs. The Hungarian Algorithm. The optimal assignment problem. The Kuhn-Munkrej Algorithm.
5. Networks : Max-Min Theorem, Separating sets, Menger's Theorem.
6. Ramsey Theory : Party Problem, relations among *Ramsey numbers*.

Prescribed Text Book

John Clerk and Derek Allan Holton : A first look at Graph Theory (Allied Publishers Ltd./World Scientific).
Chapter 1 : Sections 1.1 to 1.8, Chapter 2 : Sections 2.1

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to 2.6, Chapter 3 : Sections 3.1 to 3.4, Chapter 4 :
Sections 4.1 to 4.5, Chapter 8 : Sections 8.1, 8.3, Chapter
9 : Sections 9.1, 9.2, and 9.3.

Reference Books

1. F. Haray : Graph Theory.
2. Narsingh Deo : Introduction to Graph Theory with applications to Engineering and Computer Science.
3. Bhave and T. T. Raghunathan : Elements of Graph Theory.
4. Bondy and Murty : Graph Theory with applications.

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MT 305 : Numerical Analysis

1. Iterative solutions of Nonlinear Equations : Bisection Method, Fixed-Point iteration, Newton's method, Secant method, Acceleration of convergence, Newton's method for two nonlinear equations, Polynomial equation methods.
2. Polynomial Interpolation : The Lagrange interpolation polynomial, Divided difference interpolation, Aitken's Algorithm, Finite difference formulas, Choice of nodes and non-convergence of polynomial interpolation.
3. Systems of Linear equations : Gauss elimination with partial pivoting, Error analysis, Matrix factorization methods (Doolittle reduction, crout reduction). Iterative refinement. Iterative techniques, Gauss-Seidel iteration Acceleration and successive overrelaxation.
4. Numerical Calculus : Numerical differentiation, Forward difference Quotient, Central difference quotient, General approach to containing difference formulas, Interpolatory quadrature (order of methods), Newton-Cotes methods, Error estimates for trapezoidal rule and Simpson's rule.
5. Numerical solution of Differential Equations : Euler's method, Analysis of Euler's method, Order of Euler's method, Runge-Kutta method, One step modified and midpoint methods, Runge-Kutta methods for systems at equations.
6. The Eigenvalue Problem : Power method, Gerschgorin Disk Theorem, Eigenvalues of symmetric matrices. Jacobi method, Householder transformations.

Prescribed Text Books

1. James L. Buchanan and Peter R. Turner : Numerical Methods and Analysis (McGraw-Hill).

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2. John H. Mathews : Numerical Methods for Mathematics, Science and Engineering (Prentice-Hall) 2nd edition.

Reference Books

1. Kendall E. Atkinson : An Introduction to Numerical Analysis (John Wiley and Sons).
2. F. B. Hildbrand : Introduction to Numerical Analysis (Mc-Graw Hill - Indian Edition).
3. M. K. Jain, S. R. K. Iyengar, R. K. Jain : Numerical Methods for Scientific and Engineering Competition (Wiley Eastern).

MT 306 : Advanced Complex Analysis

0. Revision : Weierstrass theorem, Taylor's theorem, Laurent series [Chap. 5-1.1 to 1.3]
1. Mittag-Leffler's theorem for meromorphic functions. Infinite products. Canonical products, Weierstrass factorization theorem for entire function with zeros at negative integers. Legendre's duplication formula, (Chapter 5 : 2.1 to 2.4).
2. Entire Functions : Jensen's Formula, Poisson-Jensen formula, Hadamard's theorem, Relation between the genus and the order of an entire function, Normal families, Characterization on normal families, Arzela theorem.
3. The Riemann Mapping Theorem : Statement & Proof [Chapter 6-1.1].
4. Elliptic Functions, Simply periodic functions, Representation by exponentials, The Fourier development, doubly periodic functions, the period module, Unimodular transformations, The Canonical basis, General Properties of Weierstrass elliptic functions. The Weierstrass P-function. The functions $\text{Sec.}(z)$ and $\text{sec.}(z)$, Legendre's relation, The differential equation for Weierstrass elliptic function. [Chapter 7-1.1, 1.2, 2.1 to 2.4, 3.1 to 3.3]
5. Picard's Theorem-Lacunary Values [Chapter 8-3.1]

Prescribed Text-books

L. V. Ahlfors : Complex Analysis (3rd Edition).

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Reference Books

1. S. Lang : Complex Analysis
2. J. B. Conway : Functions of one complex variable
3. W. Rudin : Real and Complex Analysis.
4. H. Silverman : Complex Variables.
5. R. V. Churchill : Complex Variables
6. Moore and Hadlock : Complex Analysis

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MT 307 : Field Theory

Algebraic and transcendental extensions, algebraic closure, Splitting fields and normal extensions. Separable extensions, Finite - fields, Inseparable extensions.

Galois extensions : Examples and Application. Roots of unity, linear independence of characters. The norm and trace, cyclic extensions, Solvable and radical extensions, Abelian Kummer Theory. The equation $x^n - a = 0$.

Prescribed Text Book

S. Lang : Algebra, Addison Wesley, 3rd Edition.

Reference Books

1. M. Nagata : Theory of Fields (Marcel Dekker)
2. O. Zariski & P. Samuel : Commutative Algebra, Vol. I (Van Nostrand)
3. J. N. Hesteen : Topics in Algebra.
4. M. Artin : Algebra (Prentice-Hall).

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MT 308 : Computer Science I

1. C Fundamentals : C char set, Identifiers & Keywords, Data types, Constants, Variables & Arrays, Declarations, Expressions, Statements.
2. Operations & Expressions : Arithmetic Operators, Unary Operators, Relational & Logical Operators, Assignment Operators, Conditional Operators.
3. Data Input and Output : Getchar function, putchar function, scanf function, printf function, gets function, puts function.
4. Control Statements : While statement, Do-While statement, For Statement, Nested Loops, If-Else statement, switch statement, break statement, Continue statement, comma operator, Goto statement.
5. Functions : Defining a function, Accessing a function, Passing arguments to functions, Specifying arguments, Data types, Functions Prototypes, Recursion.
6. Arrays : Defining an array, Processing an array, Passing arrays to a function, Multidimensional arrays, Arrays & Strings.
7. Pointers : Fundamentals, Pointer declarations, passing pointers to a function, pointers and one-dimensional arrays.
8. Structures : Defining a structure, processing a structure, passing structures to a function.
9. Data Files : Opening and Closing a data file, Creating a data file, Processing a data file, Unformatted data files.

Book

C. Programming by SCHAUM Series.

If a computer course is offered by the student at B. Sc./ B.A. he should not be allowed to take this (CS.I) course However he can opt. Com. Scien. II.

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MT 401 : Commutative Algebra

0. Revision : Rings, Homomorphisms.
1. Nil radical, Jacobson radical, Operations on ideals, Extensions and contractions; Nakayama Lemma.
2. Modules : Free module, Finitely generated module Exact Sequences, Tensor Product of modules, Exactness properties of tensor product.
3. Rings and modules of fractions, Local Properties; Extended and contracted ideals in rings of fractions.
4. Primary decomposition, Integral dependence; Going up and Going down Theorems. Valuation rings.
5. Chain conditions, Noetherian rings. Primary decompositions of Noetherian rings, Artin rings.
6. Discrete valuation rings. Dedekind Domains an introduction.

Prescribed Text Book

M. F. Atiyah and J. G. Macdonald : Introduction to Commutative Algebra [Chapters 1 to 9].

Reference Books

1. Matsumura : Commutative Algebra
2. I. Kopsknky : Commutative rings.
3. N. Jacobson : Basic Algebra, Vol. 2.
4. N. S. Gopalkrishan : Commutative Algebra.

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MT 402 : Hydrodynamics

1. Lagrangian and Eulerian methods. Stream lines, Stream tubes, equation of continuity, irrotational and rotational motion, circulation. Euler's dynamical equations, surface conditions. Velocity potential, Bernoulli's theorem.
2. Motion in two dimensions, stream function. Use of complex potential for irrotational flow. Circle theorem, uniqueness theorem, Kinetic energy of an infinite mass of fluid, constancy of circulation, flow past a moving cylinder, Biot-Savart theorem, theorem of Kutta and Joukowski, Rectilinear vortices, vortex pairs, Kármán vortex stream.
3. Axi-Symmetric fluid motion, Stokes' stream function, flow past a solid of revolution.
4. Viscosity, Most general motion of a fluid element, strain quadric, stress quadric. Relation between stress and rate of strain components.

Prescribed Text Books

1. F. Chorlton : Text Book of Fluid Dynamics.
2. D. E. Rutherford : Fluid Mechanics.
3. J. L. Bansal : Viscous Fluid Dynamics.

Reference Books

1. Milne-Thomson : Theoretical Hydrodynamics.
2. G. K. Batchelor : An Introduction to Fluid Dynamics.
3. N. Curle and H. J. Davies : Modern Fluid Dynamics, Vol. I.
4. R. C. Binder : Advanced Fluid Mechanics.
5. Landau and Lifshitz : Fluid Mechanics.

MT 403 : Lattice Theory

1. Two definitions of lattices, Representation of finite poset by covering relations, Hasse diagrams, homo-morphism, Isotone maps, ideals, convex sublattice, congruence relations, Congruence lattices, The homomorphism theorem, Product of lattices, Complete lattices, Ideal lattice, Distributive-modular inequalities and identities, Complements, Pseudocomplements, Boolean lattices, Boolean lattice of Pseudo-complements in a meet semilattice, Atoms, Irreducibility of elements. [Chapter 1- Sections 1, 2, 3, 4, 6.]
2. Characterization theorem for modular and distributive lattices, Dedekind's characterization of modular lattices, Birkhoff's characterization of distributive lattices, Representation of distributive lattices, Stone theorems, Natchbin theorem, Hashimoto's theorem, Distributive lattices with pseudocomplementation, Stone lattices, Characterization of stone lattices, Stone Algebra, Characterization of stone Algebra. [Chapter 2 - Section 1.6]
3. Distributive, Standard and Neutral elements; Distributive, standard and neutral ideals, Structure theorems. [Chapter 3 - Sections 2, 3, 4].

Prescribed Text

George Gruatzer : General Lattice Theory

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MT 404 : Combinatorics

1. Permutations and Combinations, Application to Probability.
2. The Principle of Inclusion and Exclusion, Mobious Inversion. Partially Ordered sets and their mobious functions.
3. Generating Functions and Recursions.
4. Partitions, Identities and Arithmetic Properties, Guass-Jacobi identity, Jacobi identity, Asymptotic Properties of $P(n)$.
5. Distinct Representatives : The Theorems of P. Hall and D. Konig, Simultaneous representatives, The Permanent Proof of the Van der Waerden conjecture, Permanents of Integral Matrices with Constant Line Sum.
6. Ramsey's Theorem : Statement of the Theorem. Application of Ramsey's Theorem.
7. Hadamard Matrices : Paley's Constructions, Williamson's method, An infinite class of Williamson's Matrices. Three Recent Methods.

Prescribed Text Book

Marshall Hall Jr. : Combinatorial Theory, 2nd Edition (Wiley-Inter Science Publications).

Reference Books

1. John Riordan : An Introduction to Combinatorial Analysis (Wiley Publications).
2. Gerald Berman and K. D. Fryer : Introduction to Combinatorics (Academic Press).
3. V. K. Balakrishnan : Schaum's outline series Theory and Problems of Combinatorics (Mc-Graw Hill).

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4. Alan Tucker : Applied Combinatorics, 3rd Edition (John Wiley & Sons).
5. K. D. Joshi : Foundations of Discrete Mathematics.

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MT 405 : Several Complex Variables

Elementary properties of functions of several complex variables : Analytic continuation; Sub-harmonic functions; Hartog's Theorem; Singularity of Holomorphic functions.

Prescribed Text Book

Raghavan Narasimhan : Several Complex Variables [Chicago Lectures in Mathematics Series (1971)] Chapters 1 to 4.

Reference Books

1. B. Malgrange : Lectures on functions of Several Complex variables.
2. R. Guinng and R. Rossi : Analytic functions of Several Complex Variables.

MT 406 : Banach Algebras and Spectral Theory

1. Definition of Banach Algebra. Left-Continuity, right continuity of multiplication in a complex algebra. Complex homomorphism. Continuity of Complex homomorphism. Gleason-Kahane-Zela-Sko theorem that characterizes complex homomorphisms. Basic properties of spectra in Banach algebra. Non-emptiness and compactness of the spectrum. Spectral radius formula. Gelfand-Mazur Theorem and its variations. Algebra of complex holomorphic function on an open set in the complex plane \mathbb{C} . Symbolic Calculus and its applications. multiplicative group of all invertible elements of a Banach algebra.
2. Ideals and homomorphisms in a commutative Banach algebra. The Maximal ideal space of a commutative Banach algebra. Semisimple case. Involution on Banach algebras. B^* -algebras. Gelfand-Naimark theorem. Square roots of hermitian elements. Centralizer in a Banach algebra. Study of spectra of hermitian, normal and positive elements in a B^* -algebra, Positive functionals and their properties. Characterizations of extreme point of the set of all positive functionals and their properties. Characterizations of extreme point of set of all positive functionals on a commutative Banach algebra with involution.
3. Revision in Hilbert spaces of the concepts of orthogonal elements, triangle inequality etc. Existence of unique element of the minimal norm for a nonempty closed convex subset of a Hilbert space. Orthogonal decomposition of a Hilbert space and related results. Bounded operators and their adjoints. Normal, self-

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adjoint, unitary operators and their properties and characterizations. Fuglede-Putnam-Rosenblum commutativity theorem, Resolution of the identity E . The Banach algebra $L(E)$. The spectral Theorem. Invariant subspaces.

Lomomosec's invariant subspace theorem. Eigenvalues of normal operators. Compactness of normal operator and related properties. Positive operators and their square roots, Polar decomposition, Similarity and unitary equivalence of operators.

A characterization of a B^* -algebra. An ergodic theorem due to Beumann.

Prescribed Text Book

W. Rudin : Functional Analysis [Mc-Graw Hill] International Edition 1991, Chapters 10, 11 and 12.

Reference Books

1. G. F. Simmons : Topology and Modern Analysis [Mc-Graw Hill International Edition].
2. I. M., Gelfand D. Raikar and G. E. Shilov : Commutative Normed Rings, Chesea (1964).
3. M. A. Naimark : Normed rings.
4. C. E. Ricart : General Theory of Banach Algebras [Van Nostand] (1960).
5. R. G. Douglas : Banach Algebra Techniques in Operator Theory [Academic Press] (1972).

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MT 407 : Mathematical Analysis

1. The Fourier Transform, Integral transforms, Fourier kernels, Fourier's integral theorem. Laplace transform. foundations of operator calculus. Mellin transform. Multiple Fourier transforms.
2. Hankel Transforms, Hankel inversion theorem. Parseval's theorem for Hankel transforms. Hankel transforms of derivatives of a function. Relation between Hankel transforms and Fourier transforms. Dual integral equations.
3. Finite Transforms Finite Sine and cosine transforms, Faltung theorems for finite Fourier transforms. Multiple finite transforms. Finite Fourier transforms. Multiple finite transforms. Finite Hankel transforms. Inversion theorem for finite Hankel transforms.

Prescribed Book

I. N. Sneddon : Fourier Transforms (McGraw Hill).

Reference Books

1. I. N. Sneddon : The use of Integral Transform (McGraw- Hill)
2. Bracemell : Fourier Transforms and Its applications.

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MT 408 : Computer Science II

1. Principles of Object Oriented Programming : Basic Concepts, Benefits of OOP, Applications of OOP.
2. Tour of C++ : Structure of a C++ program, a simple C++ program, an example with class.
3. Tokens, Expressions and Control structures : Tokens, Keywords, Identifiers, Basic data types, user defined, data types, Derived data types, constants, Reference variables, Scope resolution operator, manipulators, operator overloading.
4. Functions : Function prototyping, Call by reference, Return by reference, Inline functions, Function overloading.
5. Classes and Objects : Class, Member functions, A C++ Program with class, Nesting of member functions, private Member functions, Arrays with a class. Arrays of objects, Objects as function arguments, Returning objects, Constructors and Destructors, Friends to a Class.
6. Operator Overloading:Defining Operator overloading, overloading Unary operators, overloading Binary operators, Rules for overloading operators.
7. Pointers : Introduction, Pointers to objects, this pointer, pointers to classes.
8. Inheritance : Defining derived Classes, Single Inheritance, making a private member inheritable, Multiple Inheritance.
9. Working with Files : Introduction, Opening and closing a file, Detecting End-of-File, Files modes, Files pointers, Sequential input and output operations, Updating a file, Command-Line-Arguments.

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Reference Book

Title : Object-Oriented Programming with C++

Author : E Balagurusamy

Publisher : Tata Mc-Graw-Hill.
