M.A./M. Sc. (Mathematics) - I

(For the Colleges Affiliated under Pune University)

(Revised Syllabus to be implemented sequentially from June 2008 onwards i.e. from the academic year 2008 - 09 for M. Sc. Part-I & from the academic year 2009-10 for M. Sc. Part-II)

M. Sc. Part - I			
Semester I		**Semester II**	
Course Code	**Course Title**	**Course Code**	**Course Title**
MT-501	Real Analysis	MT-601	General Topology
MT-502	Advanced Calculus	MT- 602	Differential Geometry
MT-503	Linear Algebra	MT –603	Groups and Rings
MT-504	Number Theory	MT- 604	Complex Analysis
MT-505	Ordinary Differential Equations	MT- 605 OR MT- 606	Partial Differential Equations OR Object Oriented Programming using C++

General notes:

1. Each semester comprises of five courses, each of 100 marks (80 external university examination + 20 internal assessment), except Course MT-606.
2. For first semesters all courses are compulsory.
3. For second semester student has to offer courses MT-601, MT-602, MT-603, MT-604 compulsory and one of the courses from MT-605 & MT-606. However, external student has to offer MT-605 compulsorily. i.e. external student shall not offer course MT-606. For course MT-606, there shall be external university examination for 50 marks and of two hours durations and internal examination of 50 marks (including assignments & semester end practical examination).
4. For external students, there shall be no departmental courses. i.e. external student has to offer all five courses from university courses in semester III,&IV.

MT-501: Real Analysis

1. Metric Spaces, Normed Spaces, Inner Product Spaces:
   Definitions and examples, Sequence Spaces, Function Spaces, Dimension.
2. Topology of Metric Spaces:
   Open, Closed and Compact Sets, the Heine-Borel and Ascoli-Arzela' Theorems, Separability, Banach and Hilbert Spaces.
3. Measure and Integration :
   Lebesgue Measure on Euclidean Space, Measurable and Lebesgue Integrable Functions, The Convergence Theorems, Comparison of Lebesgue Integral with Riemann Integral, General Measures and the Lebesgue L^P-Space.
4. Fourier Analysis in Hilbert Space :
   Orthonormal Sequences, Bessel's Inequality, Parseval's Theorem, Riesz-Fischer Theorem, Classical Fourier Analysis.
5. Weierstrass Approximation Theorem, Generalised Stone-Weierstrass Theorem, Baire Category Theorem and its Applications, Contraction Mapping.

Text Book: **Karen Saxe : Beginning Functional Analysis**
(Springer International Edition)
Chapters: Chapters 1 to 4 and 6.1, 6.2, 6.5

Reference Books:

1. N. L. Carothers: Real Analysis (Cambridge University Press)
2. T. M. Apostol: Mathematical Analysis (Narosa Publishing)
3. S. Kumaresan: Topology of Metric Spaces (Narosa Publishing)
4. G. F. Simmons: Introduction to Topology and Modern Analysis. (Mc-Graw Hill)
5. W. Rudin : Principles of Mathematical Analysis. (Mc-Graw Hill)

MT - 502: Advanced Calculus

1. Derivative of a scalar field with respect to a vector, Directional derivative, Gradient of a scalar field, Derivative of a vector field, Matrix form of the chain rule, Inverse function theorem and Implicit function theorem.

2. Path and line integrals, The concept of a work as a line integral, Independence of path, The first and the second fundamental theorems of calculus for line integral, Necessary condition for a vector field to be a gradient.
3. Double integrals, Applications to area and volume, Green's Theorem in the plane, Change of variables in a double integral, Transformation formula, Change of variables in an n-fold integrals.
4. The fundamental vector product, Area of a parametric surface, Surface integrals, The theorem of Stokes, The curl and divergence of a vector field, Gauss divergence theorem, Applications of the divergence theorem.

Text Book:

T. M. Apostol: Calculus vol. II (2nd edition)(John Wiley and Sons,Inc.)

Chapter 1 : Sections 81 to 8.22

Chapter 2: Sections 10.1 to 10.11 and 10.14 to 10.16

Chapter 3: Sections 11.1 to 11.5 and 11.19 to 11.22 and 11.26 to 11.34.

Chapter 4: Sections 12.1 to 12.15, 12.18 to 12.21

For Inverse function theorem, Implicit function theorem refer the book ‘Mathematical Analysis’ by T. M. Apostol.

Reference Books :

1. T. M. Apostol: Mathematical Analysis (Narosa publishing house)
2. W. Rudin: Principles of Mathematical Analysis.(Mc-Graw Hill)
3. Devinatz: Advanced Calculus

MT-503 : Linear Algebra

Revision – Matrices, Determinants, Polynomials. (Chapter 1 of the Text Book).

1. Vector Spaces
   Subspaces
   Basis and dimension
   Linear Transformations
   Quotient spaces
   Direct sum
   The matrix of a linear transformation
   Duality

2. Canonical Forms
   Eigenvalues and eigenvectors
   The minimal polynomial
   Diagonalizable and triangulable operators
   The Jordan Form
   The Rational Form

3. Inner Product Spaces
   Inner Products
   Orthogonality
   The adjoint of a linear transformation
   Unitary operators
   Self adjoint and normal operators
   Polar and singular value decomposition

4. Bilinear Forms
   Definition and examples
   The matrix of a bilinear form
   Orthogonality
   Classification of bilinear forms

Text Book: - Vivek Sahai, Vikas Bist : Linear Algebra (Narosa Publishing House).
Chapters : 2 to 5

Reference Books:

i) K. Hoffman and Ray Kunje : Linear Algebra (Prentice - Hall of India private Ltd.)

ii) M. Artin : Algebra (Prentice - Hall of India private Ltd.)

iii) A.G. Hamilton : Linear Algebra (Cambridge University Press (1989))

iv) N.S. Gopalkrishanan : University algebra (Wiley Eastern Ltd.)

v) J.S. Golan : Foundations of linear algebra (Kluwer Academic publisher (1995))

vi) Henry Helson : Linear Algebra (Hindustan Book Agency (1994))

vii) I.N. Herstein : Topics in Algebra, Second edition (Wiley Eastern Ltd.)

MT-504 : Number Theory

1. Revision :- Divisibility in integers, Division algorithm, G.C.D., L.C.M. Fundamental theorem of arithmetic. The number of primes. Mersene numbers and Fermat's numbers.
2. Congruences :- Properties of congruence relation. Resicle classes their properties Fermat's and Euler's theorems. Wilson's Theorem. The congruence $X^2 \equiv -1 \pmod{p}$ has solution iff p is the form 4n+1 where p is prime. Linear congruences of degree one. Chinese remainder Theorem.
3. Arithmetic functions : Euler function, Greatest integer function, Divisor function $\delta(n)$, Mobius function $\mu(n)$. Properties and their inter relation.
4. Quadratic Reciprocity :- Quadratic residue, Legendre's symbol, Its properties, Quadratic reciprocity law, Jacobi symbol, Its properties. Sums of Two Squares.
5. Some Diophantine Equations :
   The equation $ax + by = c$, simultaneous linear equations.
6. Algebraic Numbers :- Algebraic Numbers, Algebraic number fields. Algebraic integers, Quadratic fields. Units in Quadratic fields. Primes in Quadratic fields. Unique factorization Primes in quadratic fields having the unique factorization property.

Text Book :- Ivan Nivam & H.S. Zuckerman, An introduction to number theory (Wiley Eastern Limited)

Sections: 2.1 to 2.4, 3.1 to 3.3, 3.6, 4.1 to 4.4, 5.1, 5.2, and 9.1 to 9.9

Reference Books :-

1. T.M. Apostol, An Introduction to Analytical Number Theory (Springer International Student's Edition)
2. David M Burton, Elementary Number Theory (Universal Book Stall, New Delhi)
3. S. G. Telang, Number Theory (Tata Macgrow Hill)
4. G. H. Hardy and E. M. Wright, Introduction to Number Theory (The English language book society and oxford university press)

MT-505 : Ordinary Differential Equations

Review : General remarks on solutions of differential equations, Families of curves, Orthogonal trajectories.

1. Second order linear equations : The general solution of the homogeneous equations, Use of a know solution to find another solution, Homogeneous equations with constant coefficients. The method of undetermined coefficient. The method of variation of parameters.
2. Qualitative Properties of solutions of ordinary differential equations of order two : Sturm separation theorem. Normal form, Standard form, Sturm's comparison theorem.
3. Power Series solutions : Review of power series, Series solutions of first order equations, Second order linear equations, Ordinary points, Regular singular points, Indicial equations, Gauss's Hypergeometric equation, The point at infinity.
4. Systems of first order equations : General remarks on systems, Linear systems, Homogenous linear systems with constant coefficient. Non-linear systems, Volterra's Prey-Predator equations.
5. Non-linear equations : Autonomous systems, Critical points, Stability, Liapunov's direct method, Nonlinear mechanics, Conservative systems.
6. The existence and uniqueness of solutions. The method of successive approximations, Picards theorem, Systems, The second order linear equations.

Text Book :

G.F. Simmons : Differential equations with applications and Historical Notes second edition (Mc-Graw Hill).

Sections : 15 to 19, 24 to 31, 54 to 63, 68 to 70.

Reference Books :

1. G. Birkhoff and G.C. Rota : Ordinary differential equations. (John Wiley and Sons)
2. E. A. Coddington : Ordinary differential equations. Prentice Hall of India.
3. S. G. Deo, V. Lakshmikantham, V. Raghvendra. Text book of ordinary Differential Equations. Second edition. Tata Mc-Graw Hill.

MT-601 : General Topology

1. Countable and uncountable sets :
   Infinite sets, the Axiom of Choice Continuum Hypothesis, Well-ordered sets, The maximum principle.
2. Topological spaces and continuous functions :
   Basis for topology, Order topology, Continuous functions, Product topology, Metric topology, Quotient topology.
3. Connectedness and compactness :
   Connected spaces, Components and local connectedness, Compact spaces, Limit point compactness, Local compactness, One point compactification.
4. Countability and Separation Axioms :
   The Countability Axioms, Separation Axioms, Normal spaces, The Urysohn
   Lemma, The Urysohn metrization theorem (statement only), The Tietze extension theorem (statement only).
5. The Tychonoff theorem, Completely regular spaces.

Text Book : J.R. Munkers : Topology A first Course (Prentice Hall of India).
Sections : 1.7, 1.9, 1.10, 1.11, 2.1 to 2.11, 3.1 to 3.8, 4.1 to 4.4, 5.1 and 5.2.

Reference Books :

1. J. Dugundji : Topology (Allyn and Bacon, Boston, 1966.)
2. K. D. Joshi : Introduction to General Topology (Wiley Eastern Limited).
3. J. L. Kelley : General Topology (Springer Verlag, New York 1991.)
4. L. A. Steen and J. A. Seebach Jr. : Counterexamples in Topology (Holt Rinehart and Winston, Inc. New York 1970.)
5. S. Willard : General Topology (Addision-Wesley Publishing company, Inc., Reading, Mass., 1970)

MT-602 : Differential Geometry

1. Graphs and Level sets. Vector fields, The Tangent space.
2. Surfaces, Vector fields on surfaces, Orientation.
3. Gauss Map, Geodesics, Parallel transport.
4. The Weingrten Map.
5. Curvature of plane curves.
6. Arc lengths and line integrals
7. Curvature of surfaces
8. Parameterized surfaces
9. Local Equivalence of surfaces and parameterized surfaces.

Text Books : -

J.A. Thorpe : Elementary Topics in Differential Geometry (Springer Verlag)

Chapters : 1 to 15.

Reference Books :-

1. B Oneill : Elementary differential Geometry (Academic - New York)
2. do Carmo M. : Differential Geometry of curves and surfaces. (Englewood Cliffs, N.J., Prentice Hall, 1976).
3. R. Millman and G. Parker : Elements of differential Geometry. (Englewood Cliffs, N.J., Prentice Hall, 1977).

MT-603 : Groups and Rings

1. Introduction to Groups, Symmetries of a square, Dihedral Groups, Examples and properties of Groups, Finite Groups, Subgroups Cyclic Groups.
2. Permutation Groups and its properties, a check-digit scheme based on D_5, Isomorphisms, Cayley's Theorems, Cosets and Langrange's Theorem, Applications of Cosets and Permutation Groups, Rotation Group of a cube and Soccer.
3. External Direct Products with Applications, Normal subgroups and Factor Groups and Applications.
4. Group Homomorphisms, First Isomorphism Theorem, Fundamental Theorem of Finite Abelian Groups.
5. Sylow Theorems, Applications of Sylow Theorems.
6. Introduction to Rings, Integral Domains, Fields, Ideals, Ring Homomorphisms, Polynomial Rings, Factorisation of Polynomials, Divisibility in Integral Domains.

Text Books :

Joseph Gallian – Contemporary Abstract Algebra (Narosa Publishing House).

Chapters :- 2 to18 and 24.

Reference Books :-

1. I.S. Luthar and I.B.S. Passi :Algebra (Volume 1) Groups (Narosa Publishing House)
2. I.N. Herstein : Topics in Algebra (Wiley - Eastern Ltd)
3. M. Artin : Algebra (Prentice Hall)
4. N.S. Gopalkrishna : University Algebra.(Wiley - Eastern Ltd)
5. Fraleigh : A First Course in Abstract Algebra
6. Herstein : Topics in Algebra (Wiley - Eastern Ltd)
7. Dummit and Foote : Abstract Algebra (Wiley - Eastern Ltd)

MT-604 : Complex Analysis

1. The extended plane and its spherical representation.
2. Elementary properties and examples of Analytic functions.

   Power series, Analytical functions, Analytical functions as mappings, Mobius transformations.
3. Complex Integration :

   Rieman - Stieltjes integrals (Theorems without proof), Power series representation of an analytical function, The index of a closed curve, Cauchy Theorem and integral formula, The homotopic version of Cauchy's Theorem and simple connectivity (Third version 6.7 without proof), Counting zero's; The open mapping theorem, Goursat's Theorem.
4. Singularities :

   Classification of singularities, Residues, The argument principle.
5. The maximum modulus theorem :

   The maximum principle, Schwarz's Lemma

Prescribed Books :

John B. Conway : Functions of one complex variable (Narosa Publishing house)

Sections : Chap. I-6, III- 1 to 3, IV- 1 to 8, V- 1 to 3, VI- 1 and 2, VII - 4.

Reference Books :-

1. Lar's V. Ahlfors : Complex Analysis (Mc Graw Hill)
2. Ruel V. Churchill / James Ward Brown : Complex Variables and Applications (McGraw Hill)

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MT-605 : Partial Differential Equations

1. **First Order P.D.E. :**

   Introduction, Charpit's Method, Jacobi's Method, Quasi-Linear Equations, Non-Linear First Order P.D.E.

2. **Second Prder P.D.E. :**

   Introduction, One Dimensional Wave Equation, Laplace Equation, Boundary Value Problems, the Cauchy Problem, Dirichlet and Neumann Problem for different regions, Harnack's Theorem, Green's Function, Heat Conduction Problem, Duhamel's Principle, Classification of P.D.E. in the case of n-variables, Families of Equipotential Surfaces, Kelvin's Inversion Theorem.

Text Book -

T. Amarnath : An Elementary Course in Partial Differential Equations (2^{nd} edition) (Narosa Publishing House)

Chapters : Chapter 1 and 2.

Reference Books :-

1. W. E. Williams : Partial Differential equations (Clarendon press-oxford)
2. E. T. Copson : Partial differential equations (Cambridge university press)
3. I.N. Sneddon :- Elements of partial differential equations (Mc-Graw Hill book company).

MT- 606 : Object Oriented Programming Using C + +

1. **Basics of C + + :**

   Structure of main( ), Data Types , Variables, Constants and keywords, Operators, Header files, printf(), scanf(), Control flow (if-else, switch, break, while, do-while, for, continue, goto and labels), Arrays, Strings , Structures and unions, Pointers.

2. **Object Oriented Concepts :**

   Methodology, Features, Advantages over Procedure Oriented Programming.

3. **Preliminaries of C + +:**

   cin, cout objects, Insertion and Extraction operators, Reference variables.

4. **Functions in C + +:**

   Function prototyping, Default arguments, Inline functions, Call by reference, Return by reference, Function overloading.

5. **Classes and Objects :**

   Access specifiers, Defining data members & member functions, Creating objects, Accessing members of a class, Array of objects, Objects as function arguments, Returning objects, Constant & Static member functions, friend function.

6. **Constructors & Destructors :**

   Basics, 'this' pointer, Types of constructor (parameterized, copy, default), Memory allocation, Destructors.

7. **Operator overloading :**

   Operator function definition, Overloading all operators, Overloading using friend functions.

8. **Inheritance :**

   Basics, Single inheritance, Private member inheritance, multiple & multilevel inheritance, overloading new & delete operators

9. **Pointers :**

   Pointers to objects , Pointers to derived class , Virtual & pure virtual functions , Command line arguments.

10. **Files :**

    Introduction, Classes for file stream operations, Opening & closing a file, 'eof' detection

*** Reference Books:**

1. Stephen Prata : C++ Primer Plus
2. Robert Lafore : Object oriented Programming using Turbo C++
3. Bruce Ekcel : Thinking in C++ Vol. 1
4. Bjarne Stroustrup : The C++ Programming Language

*** Assignments to be covered :**

1. Generate 'n' prime nos.
2. Generate 'n' perfect nos.
3. Convert the given no. to base n. Use default value n = 2.
4. Define a class " Complex" to represent complex no.

   Define following members:

   a) Necessary constructors
   b) Overload the operator + =to find sum of two complex numbers.
   c) Overload the operator - = to find subtraction of two complex numbers
   d) overload operator * = to find multiplication of two complex numbers.
   e) overload operator / = to find division of two complex numbers.
   f) overload operator + to find sum of two complex numbers
   g) overload operator - to find subtraction of two complex numbers.
   h) overload operator * to find product of 2 complex numbers.
   i) overload operator / to find division of two complex numbers.
   j) overload operators + + (Prefix & Postfix) to increment & decrement a complex numbers.
   k) overload operator ~ to find complex conjugate.
   l) overload operator << to print a complex numbers.
   m) overload operator >> to read a complex numbers.
   n) get real ( ) & get imaginary ( ), etc.

5. Define a class "vector" with following member/friend function.

   a) necessary constructors to allocate memory.
   b) Destructor.

   overload the following operators.

   c) + = to find sum of two vectors.
   d) - = to find subtraction of two vectors.
   e) * = to find product of two vectors.
   f) * = to find scalar multiplication.

g) + = to find sum of two vectors.

h) - to find subtractions of two vectors.

i) * to find product of two vectors.

j) * to find scalar multiplication.

k) + + & - - (prefix & postfix) to increment & decrement a vector.

l) << to print a vector.

m) >> to read a vector.

n) [ ] to get i^{th} element of vector.

o) overload operator = to copy vector.

p) get size ( ), resize (int n)

q) accumulate ( ), innerproduct ( ), partialsum ( ), adjacent difference ( ).

6. Define a class "matrix" with following member / friend function.

a) Necessary constructors to allocate memory.

b) Destructor. Overload the operator.

c) + = to find sum of two matrices.

d) - = to find subtraction of two matrices.

e) * = to find product of two matrices.

f) * = to find scalar multiplication.

g) + = to find sum of two matrices.

h) - = to find subtraction of two matrices.

i) * = to find product of two matrices.

j) * = to find scalar multiplication.

k) + + & - - (postfix & prefix) to inc/dec matrix.

l) << print matrix.

m) >> to read a matrix.

n) [ ] to get i^{th} element of row.

o) = to copy matrix.

p) get size ( ), resize ( ), etc.

q) inverse ( ), swap rows ( ), etc.

7. Define a class "employee" and inherit class "manager" from "employee". Define suitable constructors & destructors. Define getdata ( ) & putdata ( ) as virtual function.

8. Define class "point" to represent Cartesian co-ordinates of a point and "polar" to represent polar co-ordinates. Define suitable conversion operators to convert polar to Cartesian and vice-versa.

   Overload + and - to find addition, subtraction of two points. Allow mixed mode operations.

9. File encryption, decryption.